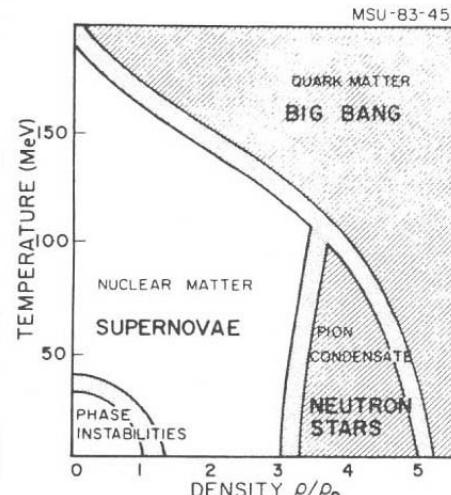
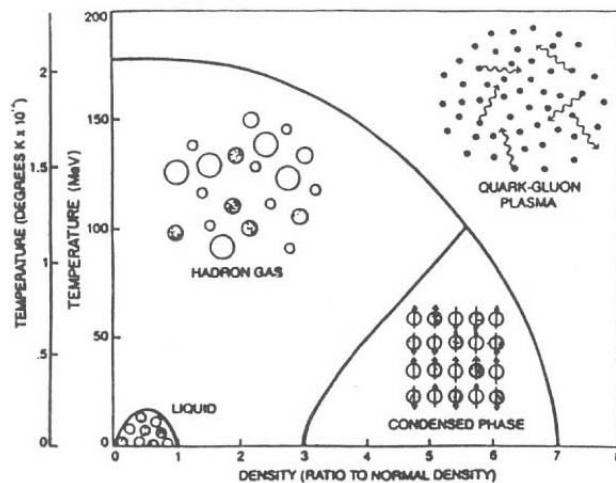
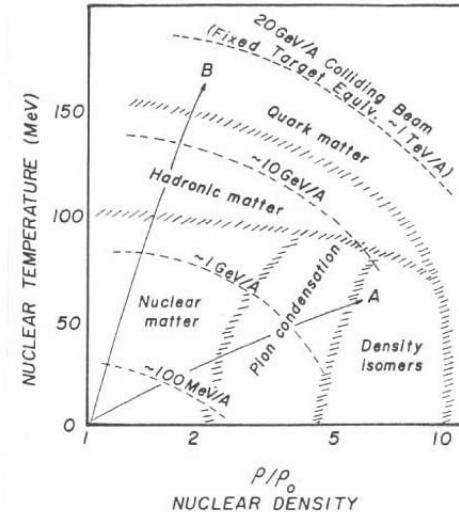
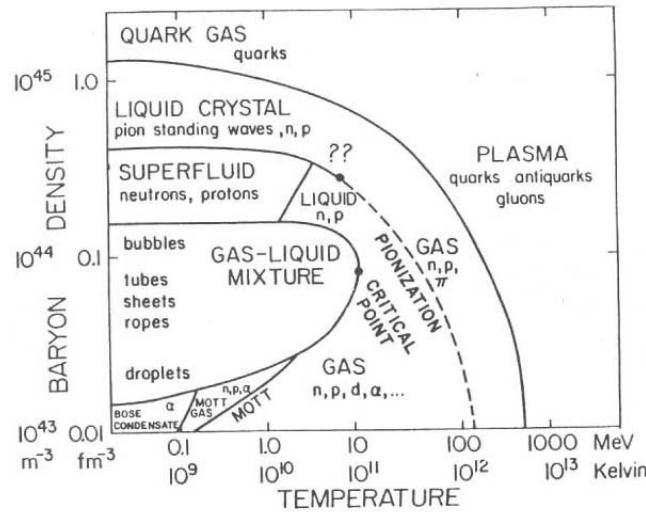
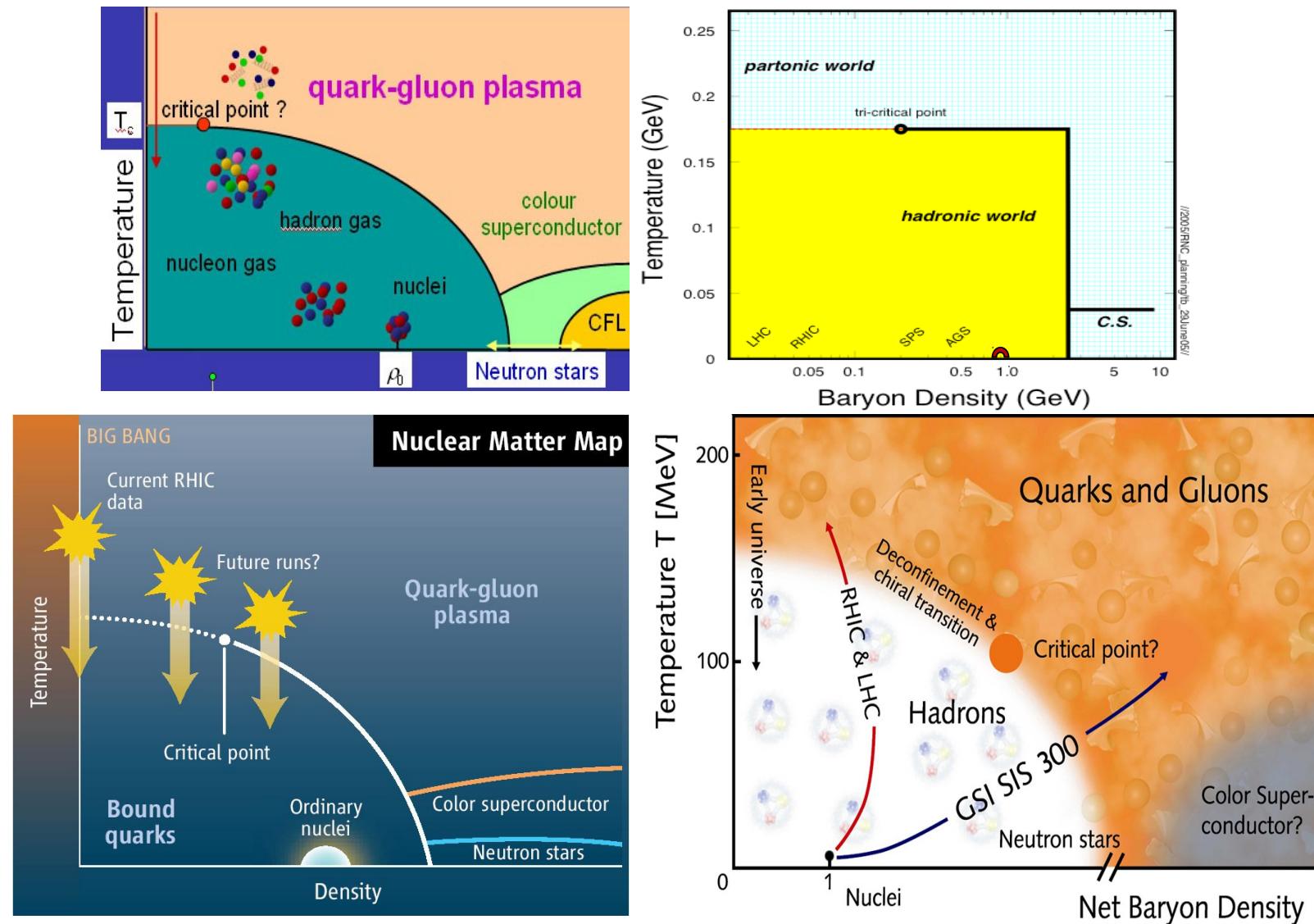


Exploring the QCD Phase Diagram



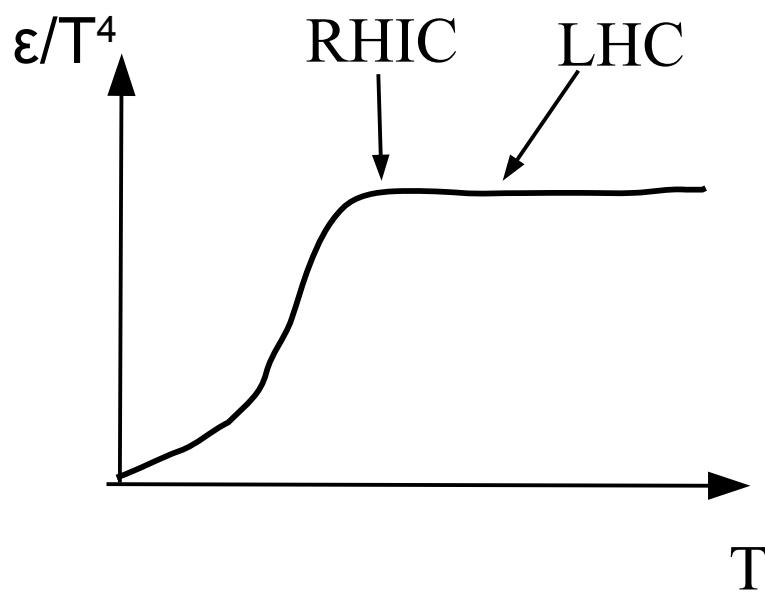
The story gets more colorful ...



Outline

- Introduction
- Some remarks about the phase diagram
- First order phase co-existence
 - Dynamical treatment
 - observables
- Remarks about some results from the BES

The Paradigm

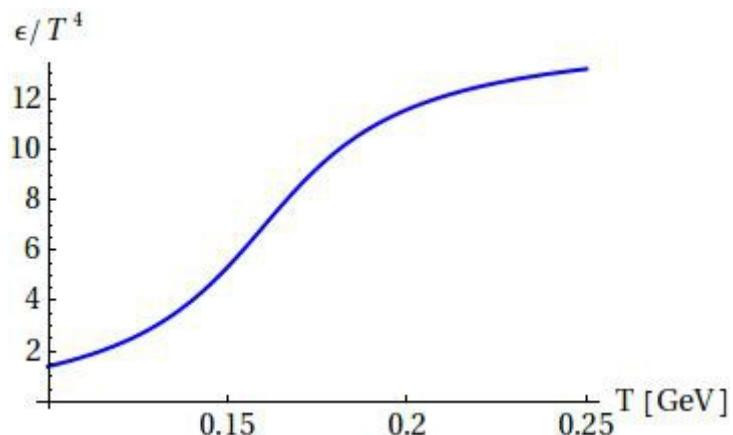


RHIC and LHC look qualitatively similar:

- Flow
- R_{AA}
- Particle production
-

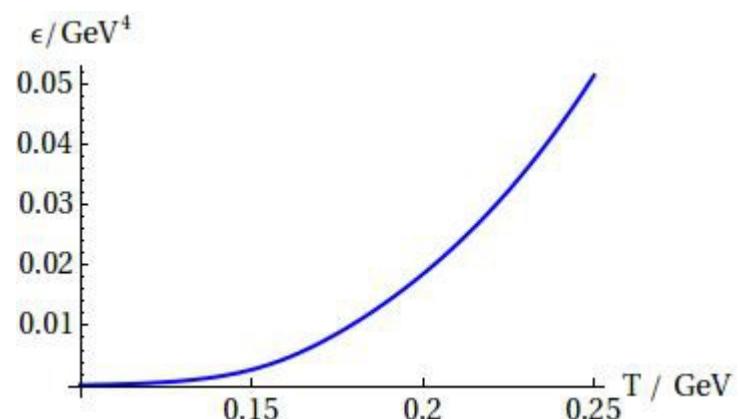
Paradigm seems in good shape
but can we establish that there is indeed a transition

The Lattice EOS



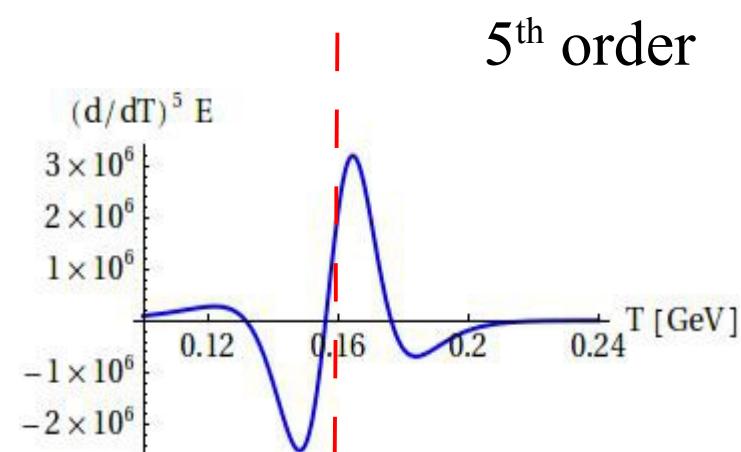
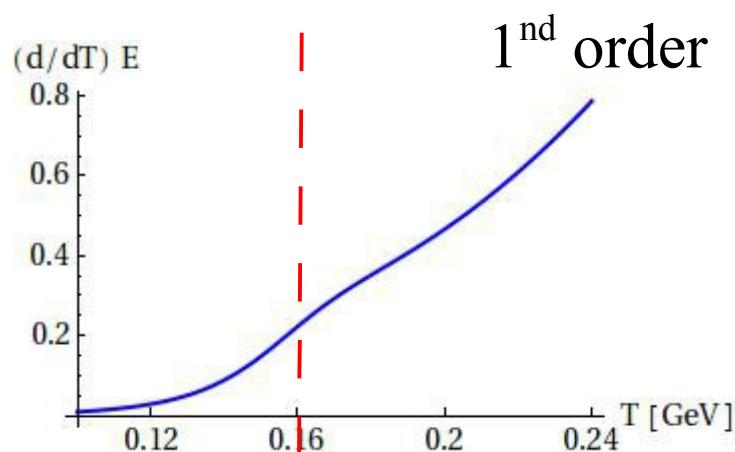
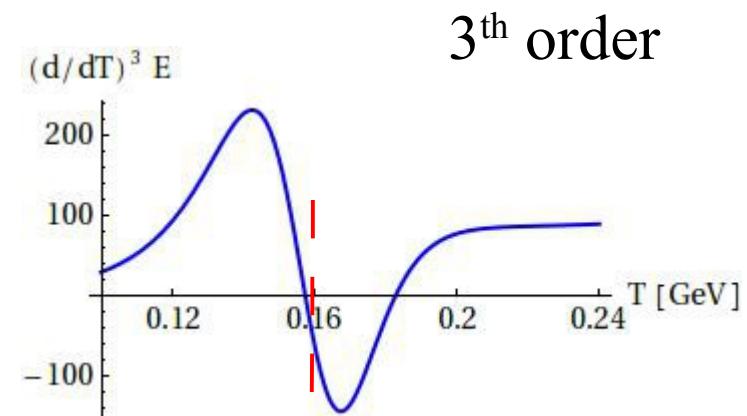
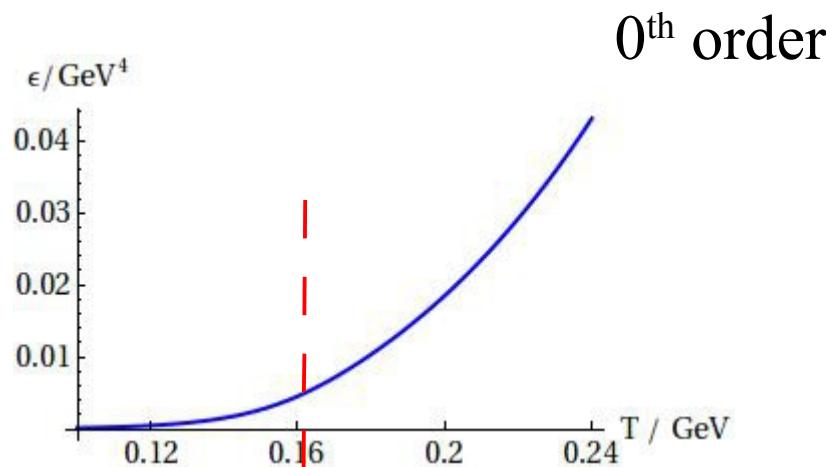
What we always see....

$$\text{“}T_c\text{”} \sim 160 \text{ MeV}$$



What it really means....

Derivatives



T_c

T_c

How to measure derivatives

$$Z = \text{tr } e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

At $\mu = 0$:

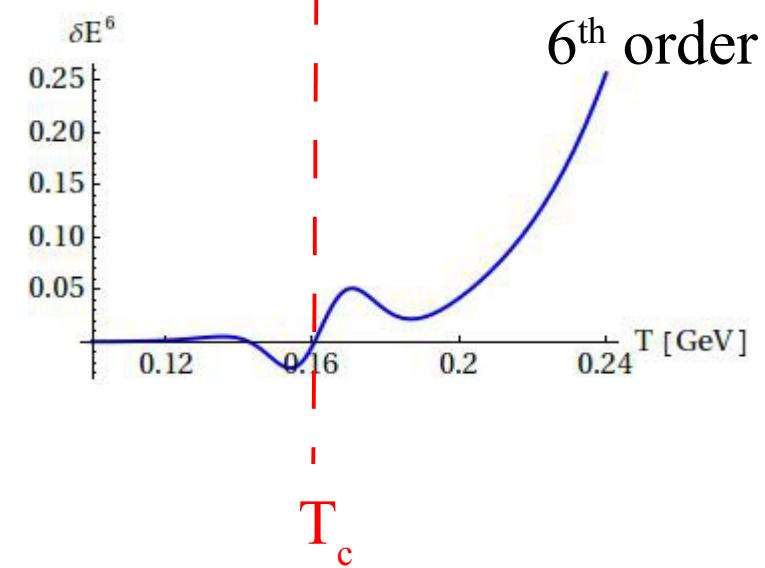
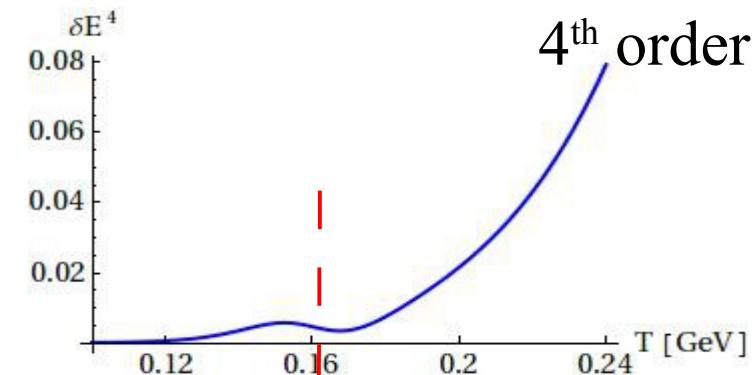
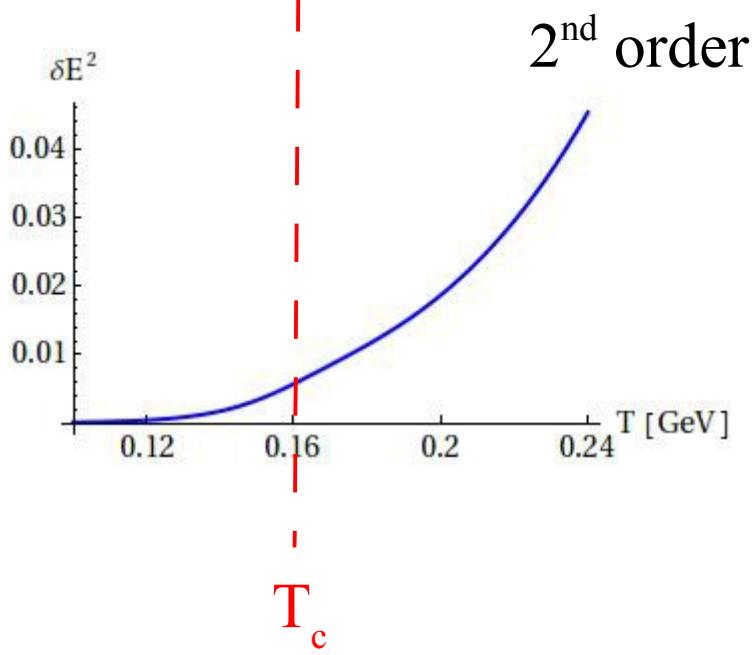
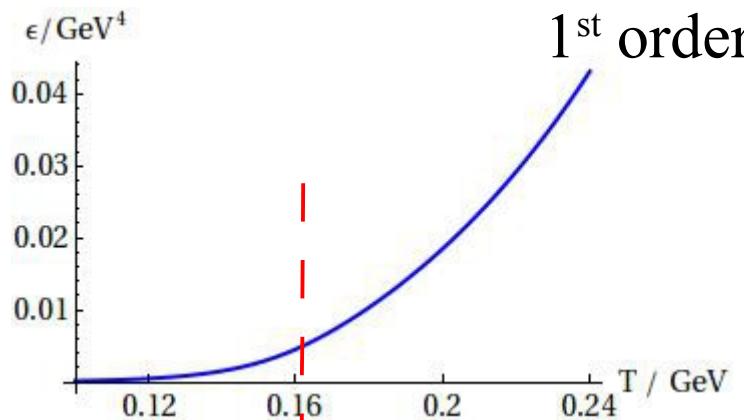
$$\langle E \rangle = \frac{1}{Z} \text{tr } \hat{E} e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the derivatives of the EOS

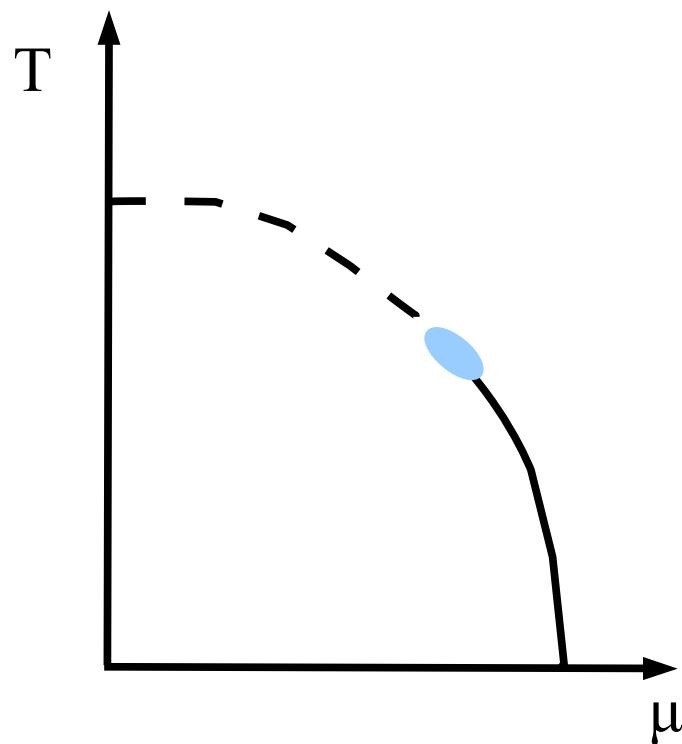
Fluctuations / Cumulants



Another way

$$F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$$

$a \sim$ curvature of critical line

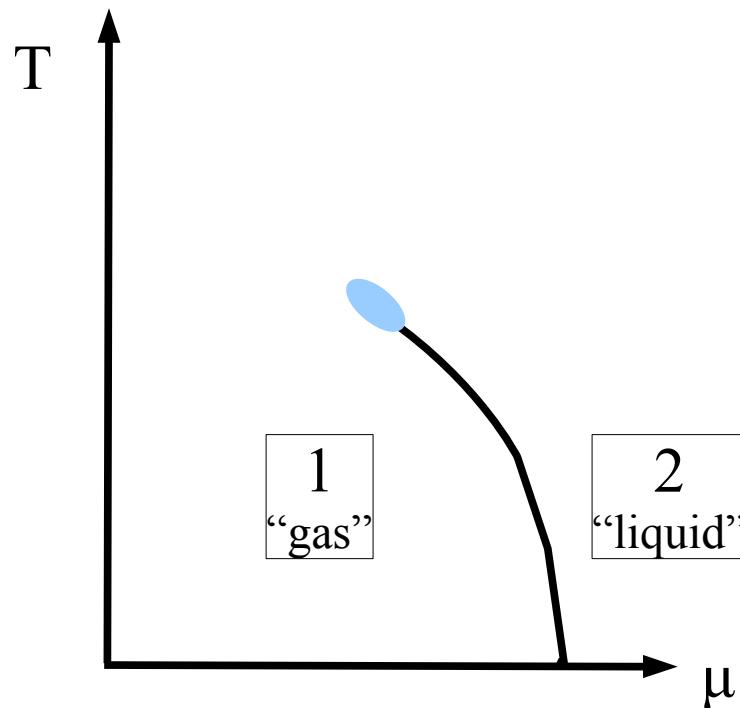


$$\partial_{\mu}^2 F(T, \mu)_{\mu=0} = \frac{a}{T} \partial_T F(T, 0)$$

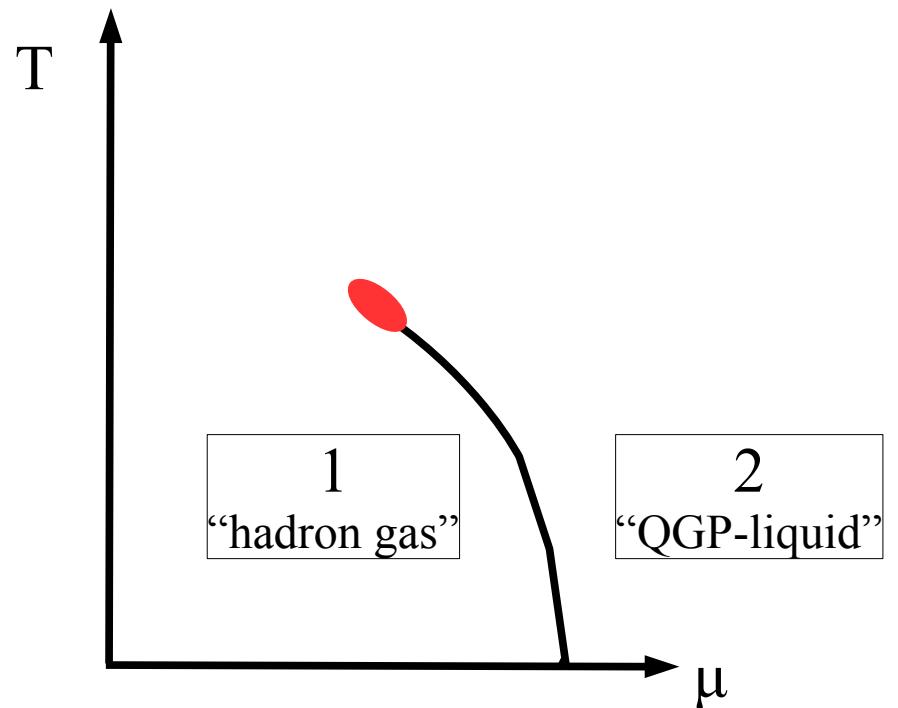
$$\partial_{\mu}^4 F(T, \mu)_{\mu=0} = 3 \frac{a^2}{T} (T \partial_T^2 - \partial_T) F(T, 0)$$

Baryon number cumulants give same info. Less problem with flow etc.

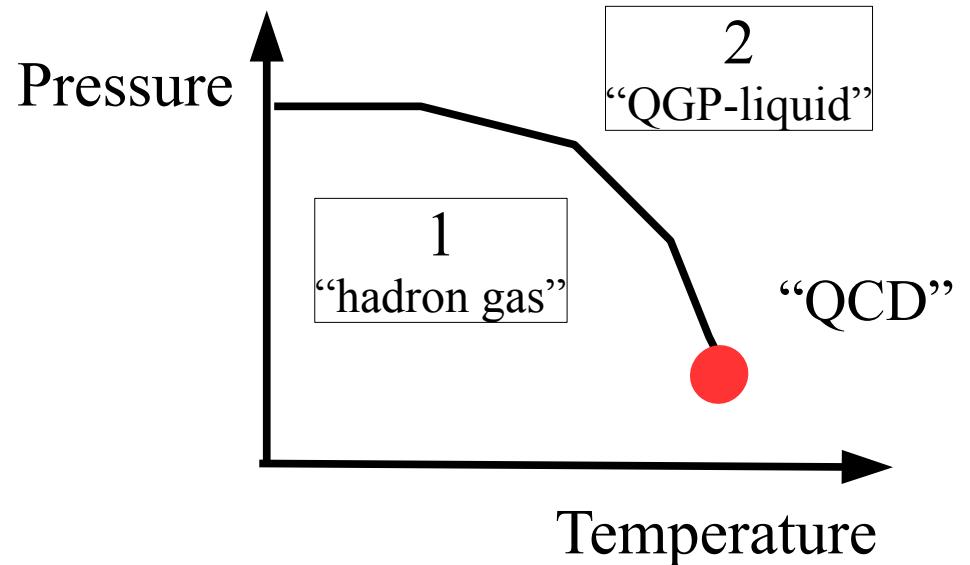
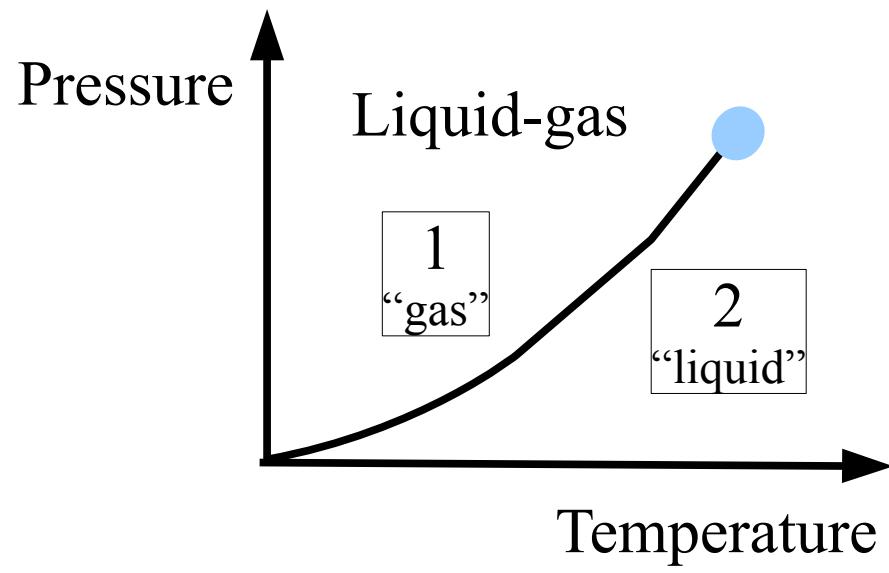
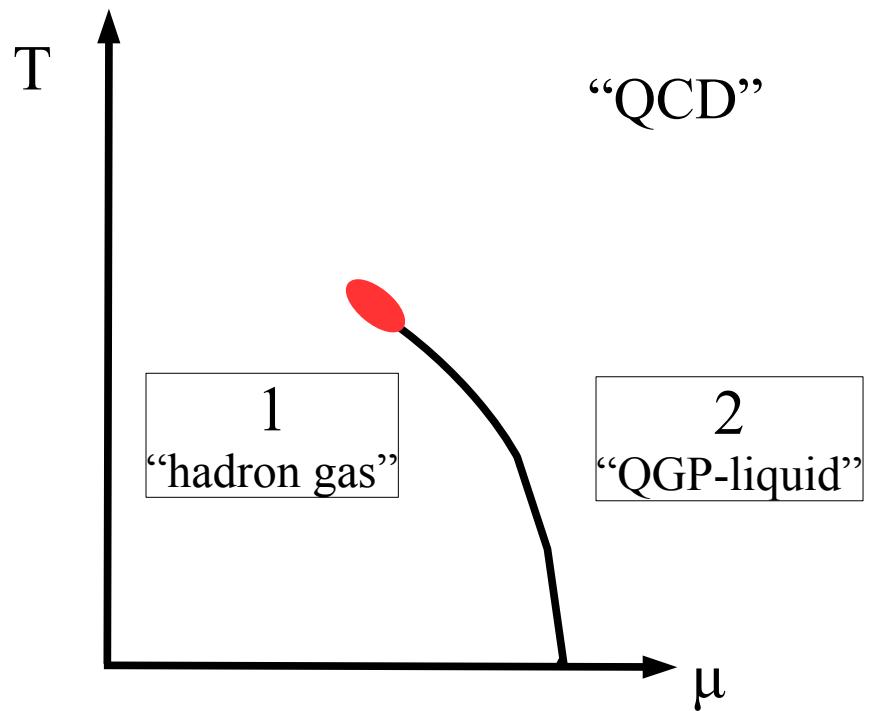
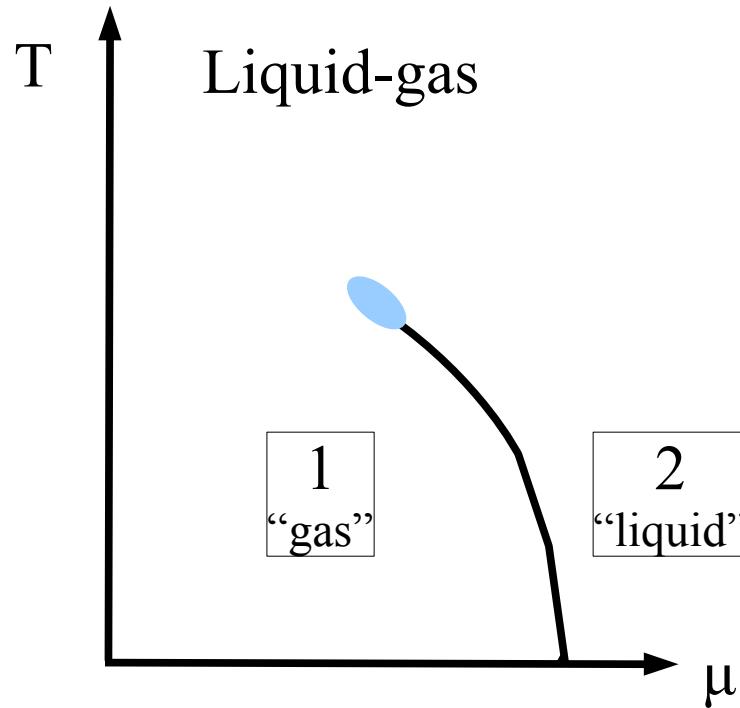
Comments on Phase diagram



Liquid-Gas
Water, nuclear matter, ...

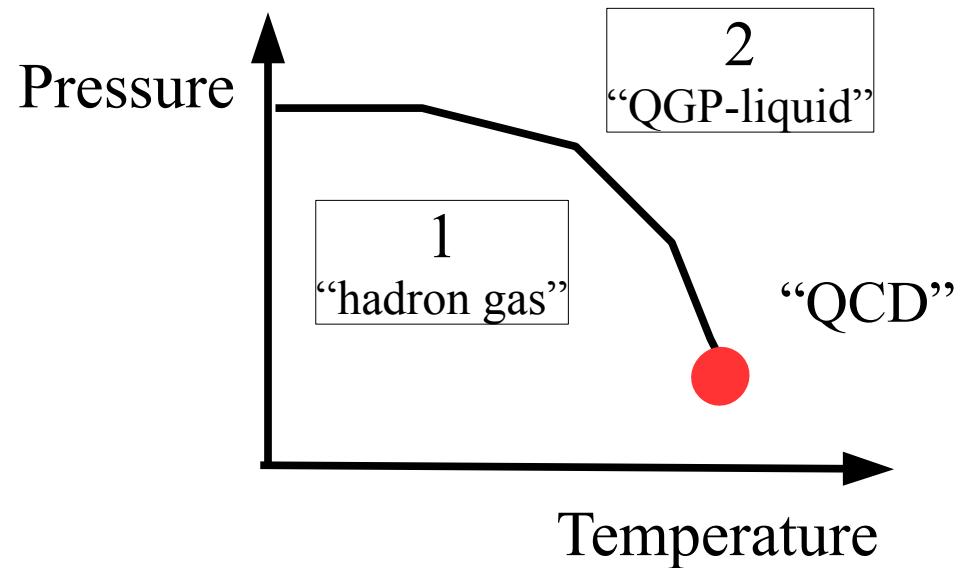
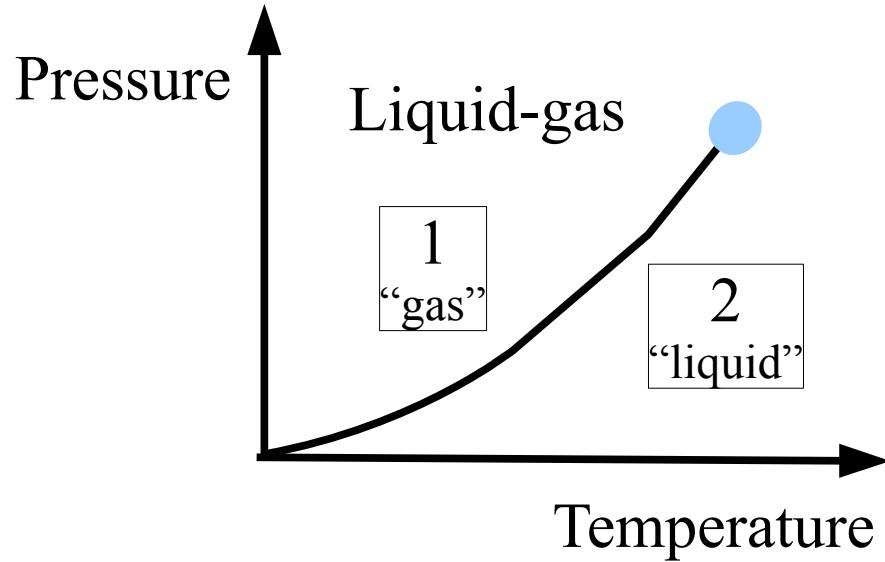


"QCD"



Difference between Liquid Gas and QCD PT

Dexheimer et al, arXiv:1302.2835



Clausius-Clapeyron: $\frac{dP}{dT} = \frac{S_1/B_1 - S_2/B_2}{1/\rho_1 - 1/\rho_2}$

$$\frac{dP}{dT} > 0 \rightarrow S_1/B_1 > S_2/B_2$$

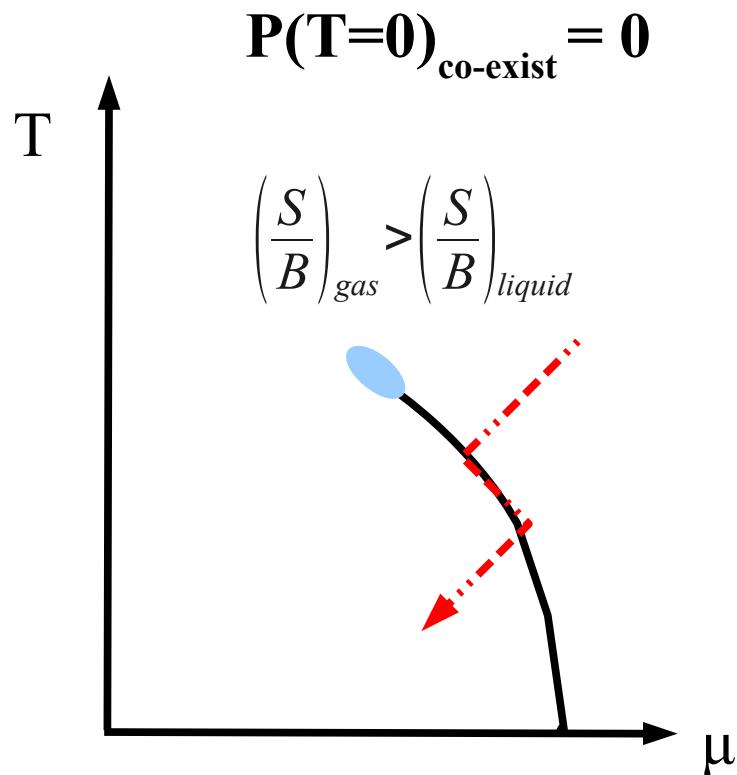
$$\left(\frac{S}{B}\right)_{gas} > \left(\frac{S}{B}\right)_{liquid}$$

$$\rho_2 > \rho_1 \rightarrow \left(1/\rho_1 - 1/\rho_2\right) > 0$$

$$\frac{dP}{dT} > 0 \rightarrow S_1/B_1 < S_2/B_2$$

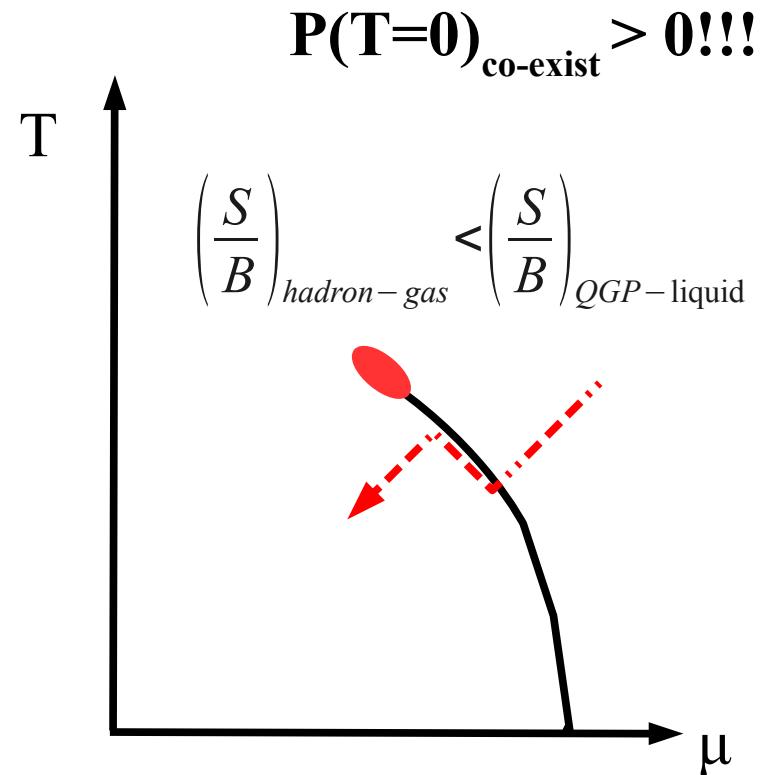
$$\left(\frac{S}{B}\right)_{hadron-gas} < \left(\frac{S}{B}\right)_{QGP-liquid}$$

Liquid-gas vs QCD



Droplets are stable in vacuum

$$\frac{dP}{dt} > 0$$

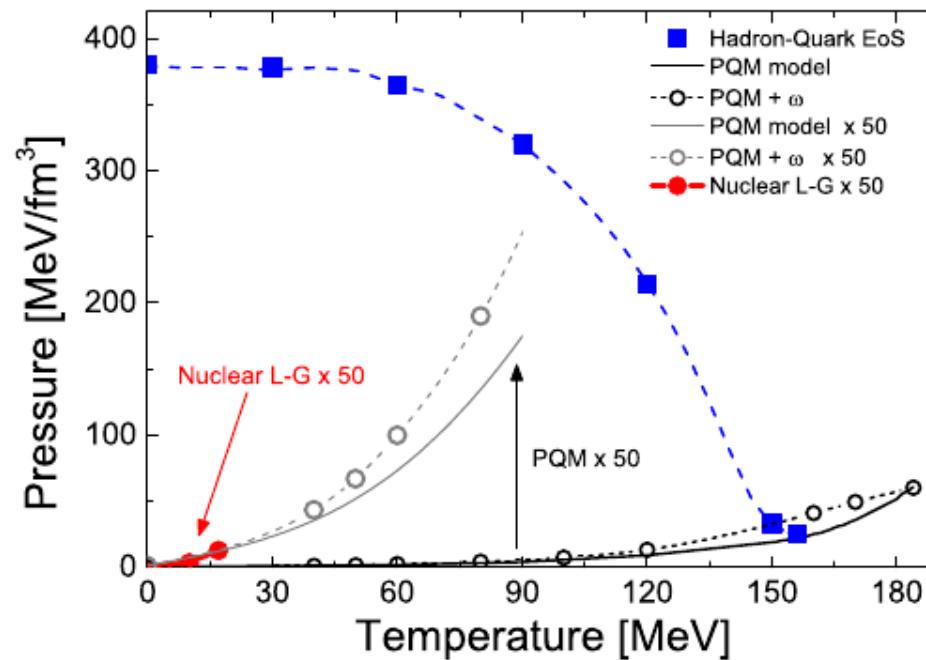


No stable droplets in vacuum

$$\frac{dP}{dt} < 0$$

Liquid-gas vs QCD

QCD: pressure at $T=T_c$ and $\mu=0$ same as at $T=0$ and $\rho \sim 2.5 \rho_0$

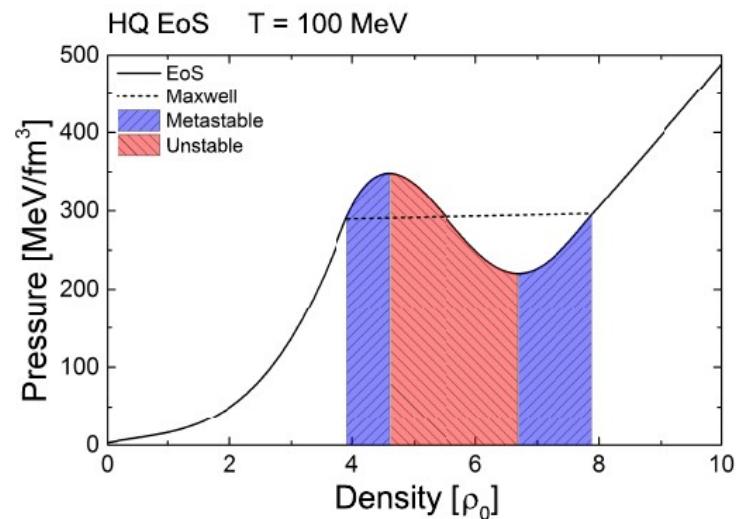
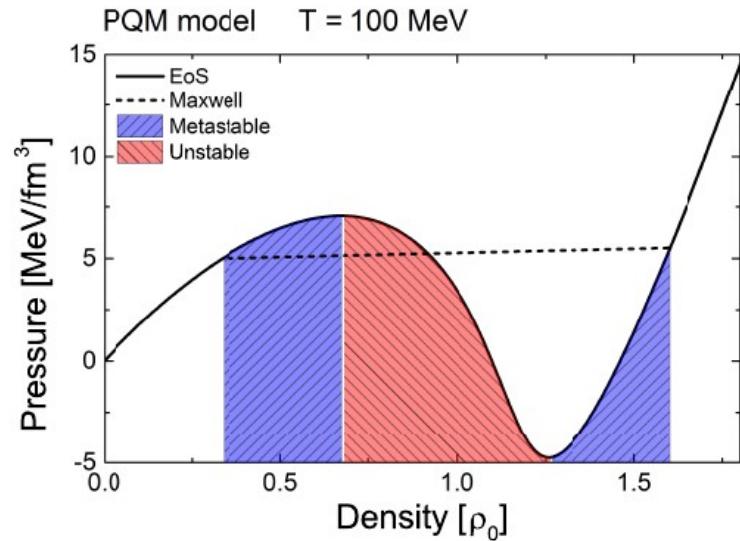


Steinheimer et al,
Phys.Rev. C89 (2014) 034901

If $T=0$ phase transition happens above $2.5 \rho_0 \rightarrow \frac{dP}{dT} < 0$

Note: virtually ALL model predicting a QCD critical point have $\frac{dP}{dT} > 0$

Liquid-gas vs QCD

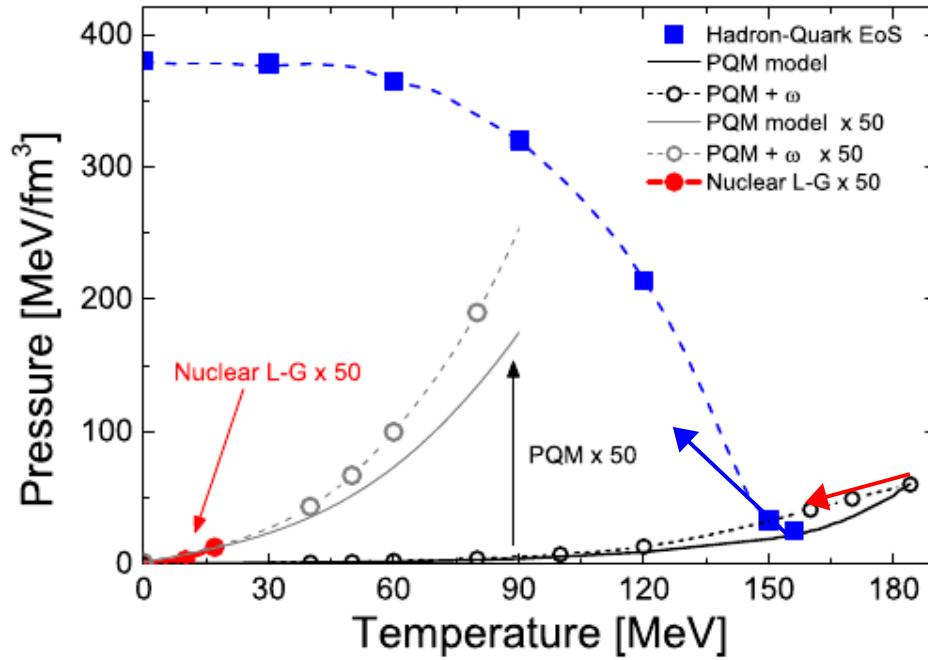


Liquid Gas:
 $T=0$: Liquid co-exists with **vacuum**

QCD:
 $T=0$: Liquid co-exists with high density nuclear matter

Steinheimer et al,
Phys.Rev. C89 (2014) 034901

Lattice to the rescue?



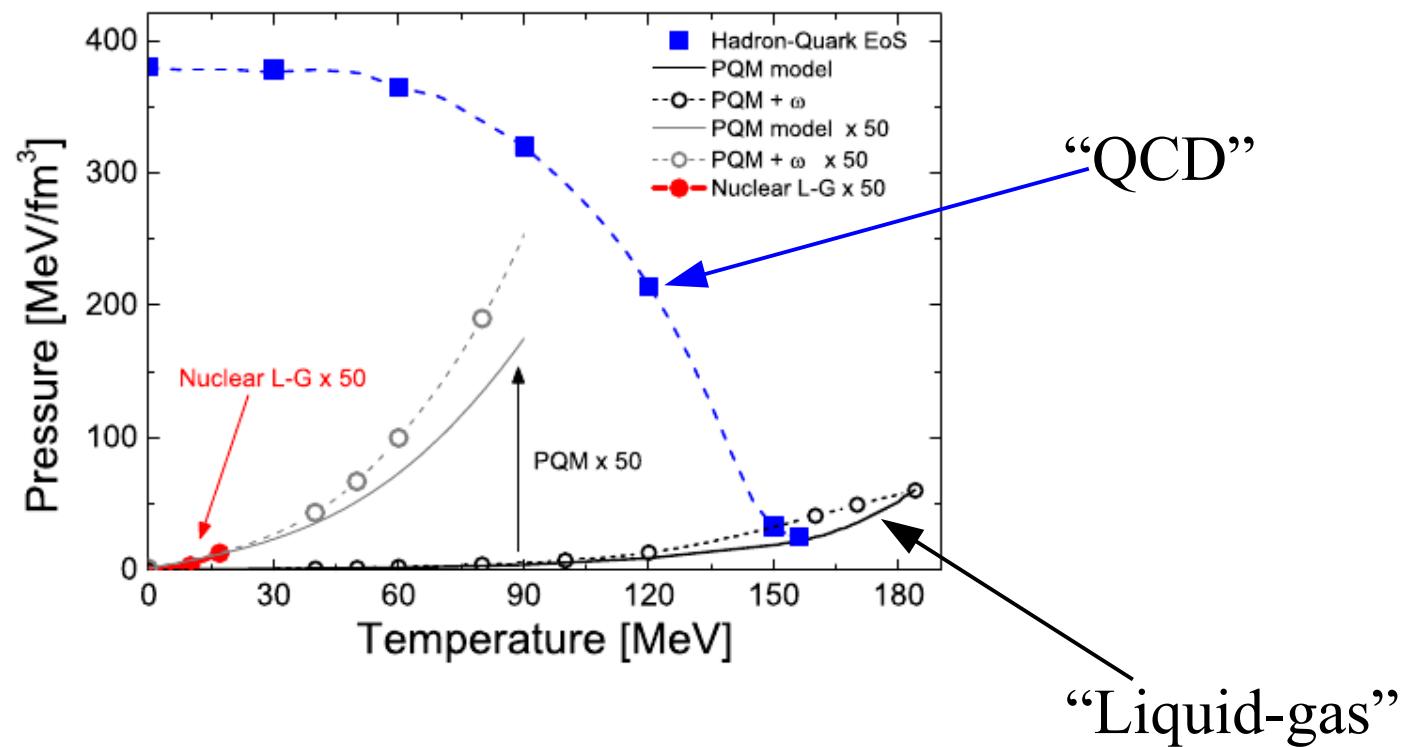
Slope of pressure
along pseudo-critical line

$$\frac{\partial}{\partial T} p_{pc}(T, \mu = 0)|_{T=T_x} = s(T_x, \mu = 0) - \frac{T_x^3}{2\kappa} \chi_2(T_x) . \quad (18)$$

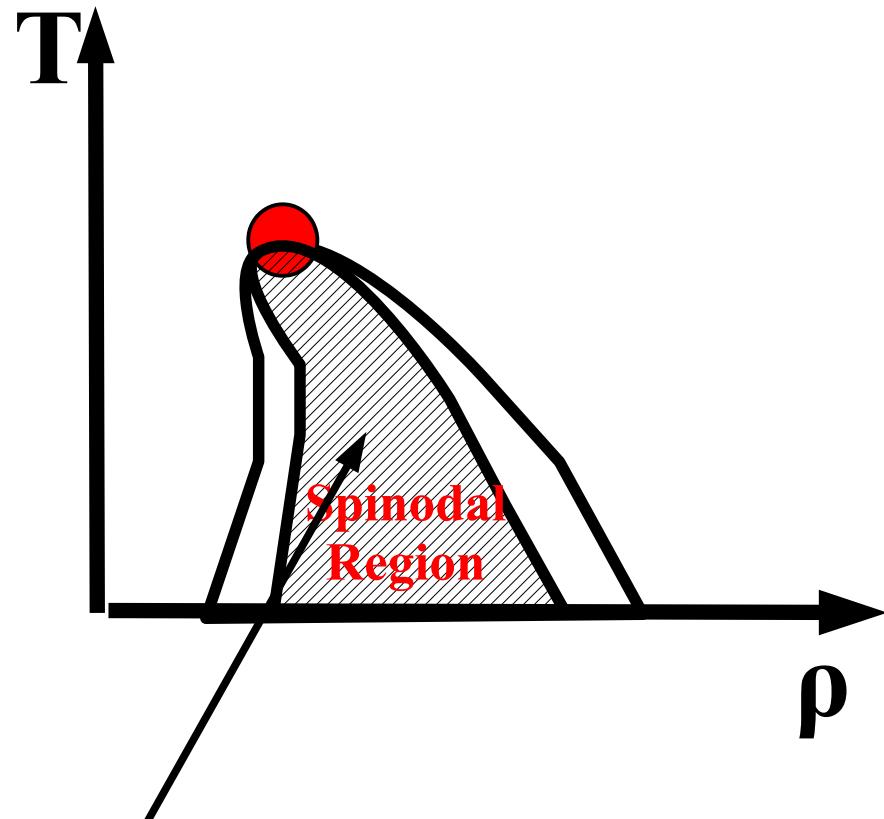
Lattice data from Wuppertal/Budapest: Sign depends on definition of
pseudo-critical line 😞

DOES IT MATTER?

Oh, YES!



Co-existence region



System should spent long time
in spinodal region

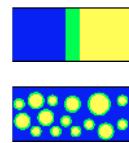
Spinodal instability:
Mechanical instability

$$\frac{\partial p}{\partial \epsilon} < 0$$

Exponential growth of clumping
Non-equilibrium phenomenon!

Phase-transition dynamics: Density clumping

Phase transition \Rightarrow Phase coexistence: surface *tension*
Phase separation: *instabilities*



Introduce a gradient term:

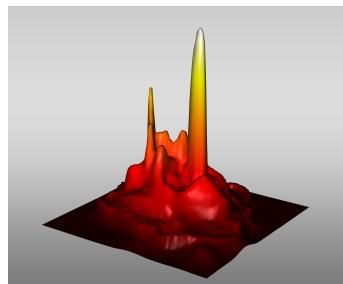
$$p(\mathbf{r}) = p_0(\varepsilon(\mathbf{r}), \rho(\mathbf{r})) - C\rho(\mathbf{r})\nabla^2\rho(\mathbf{r})$$

Insert the modified pressure into existing ideal finite-density fluid dynamics code

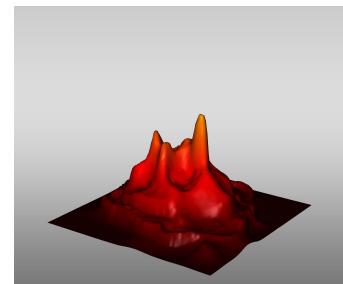
Use UrQMD for pre-equilibrium stage to obtain fluctuating initial conditions

Simulate central Pb+Pb collisions at ≈ 3 GeV/A beam kinetic energy on fixed target, using an Equation of State either with a phase transition or without (Maxwell partner):

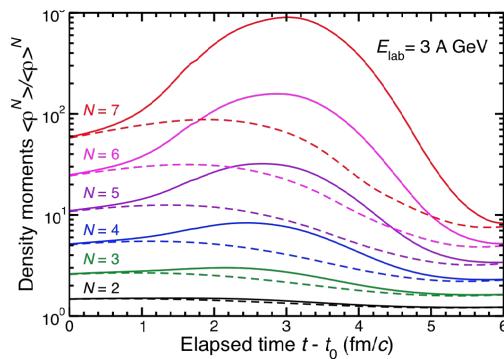
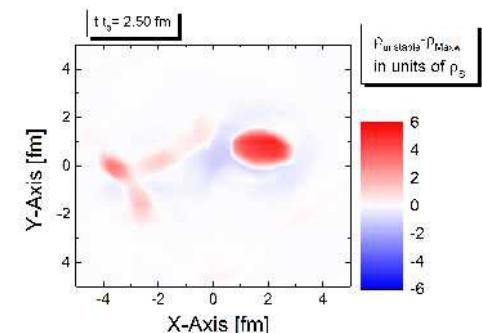
With phase transition:



Without phase transition:



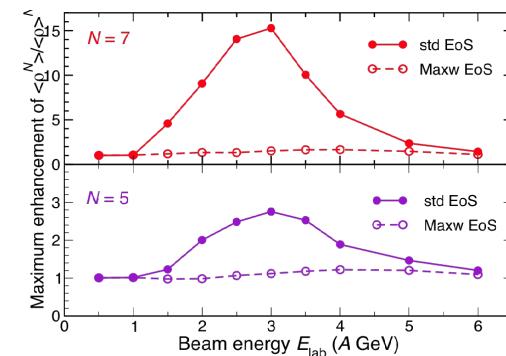
Density enhancement:

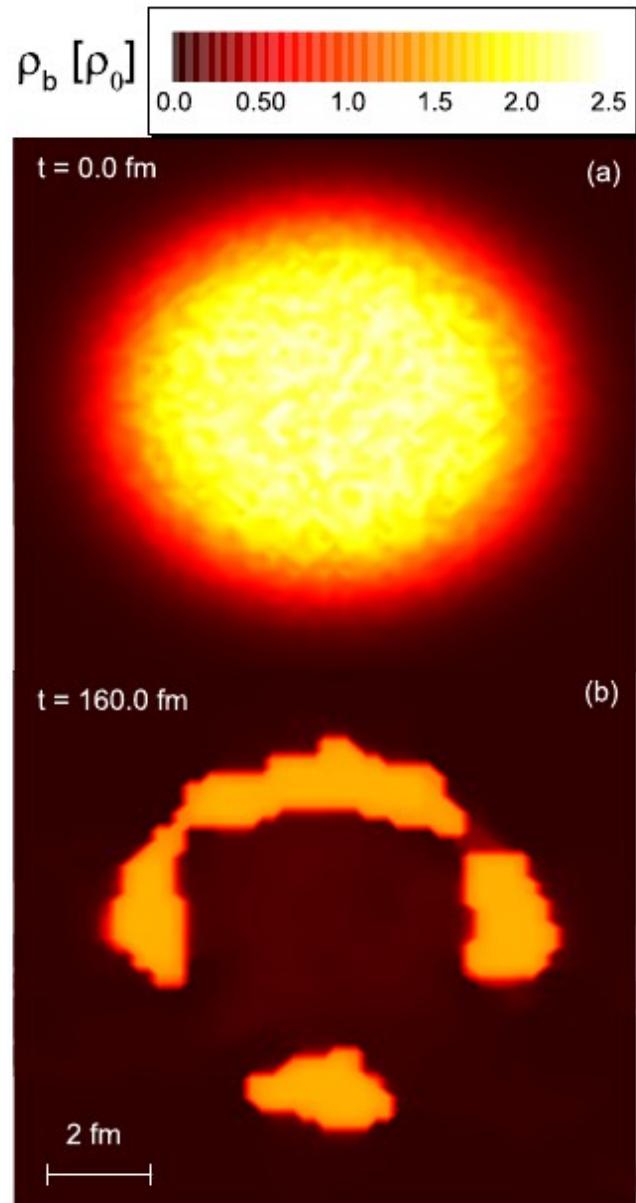


Evolution of density moments

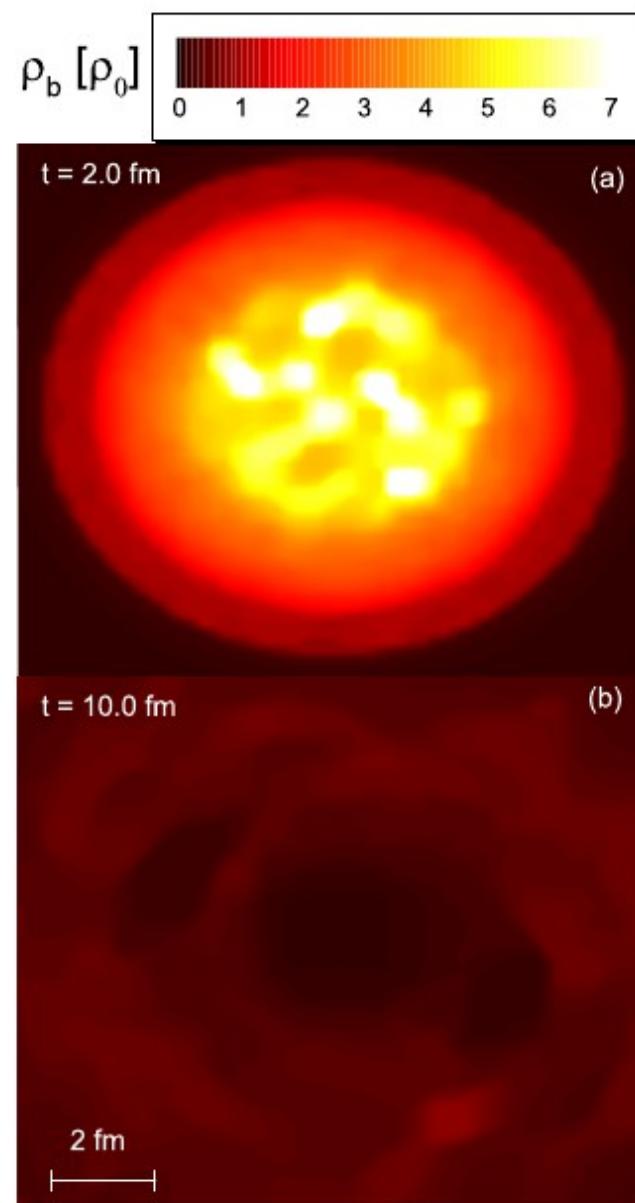
$$\langle \rho^N \rangle \equiv \frac{1}{A} \int \rho(\mathbf{r})^N \rho(\mathbf{r}) d^3 r$$

J. Steinheimer & J. Randrup,
PRL 109, 212301(2012)
PRC 87, 054903 (2013)



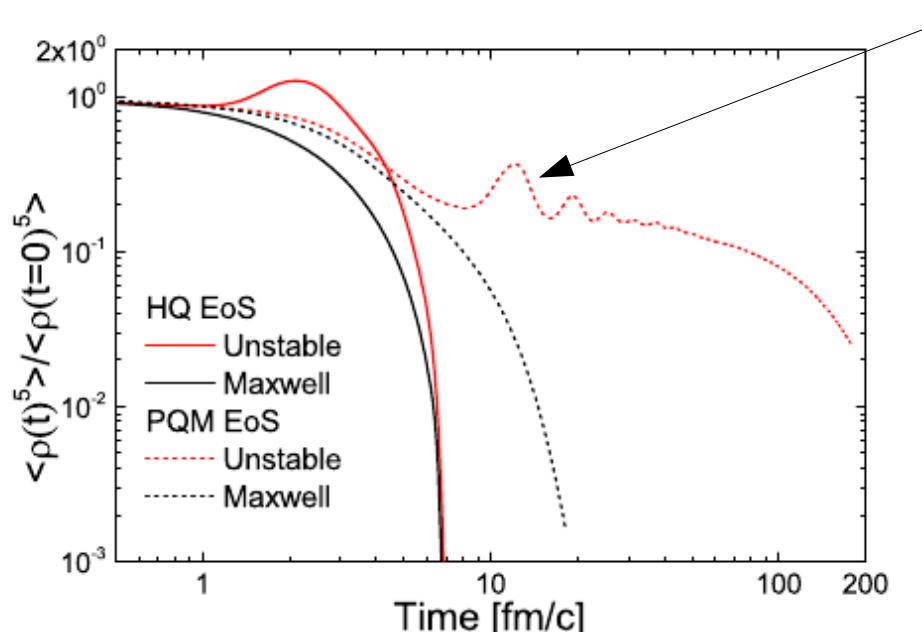


PQM (“liquid-gas”)



“QCD”

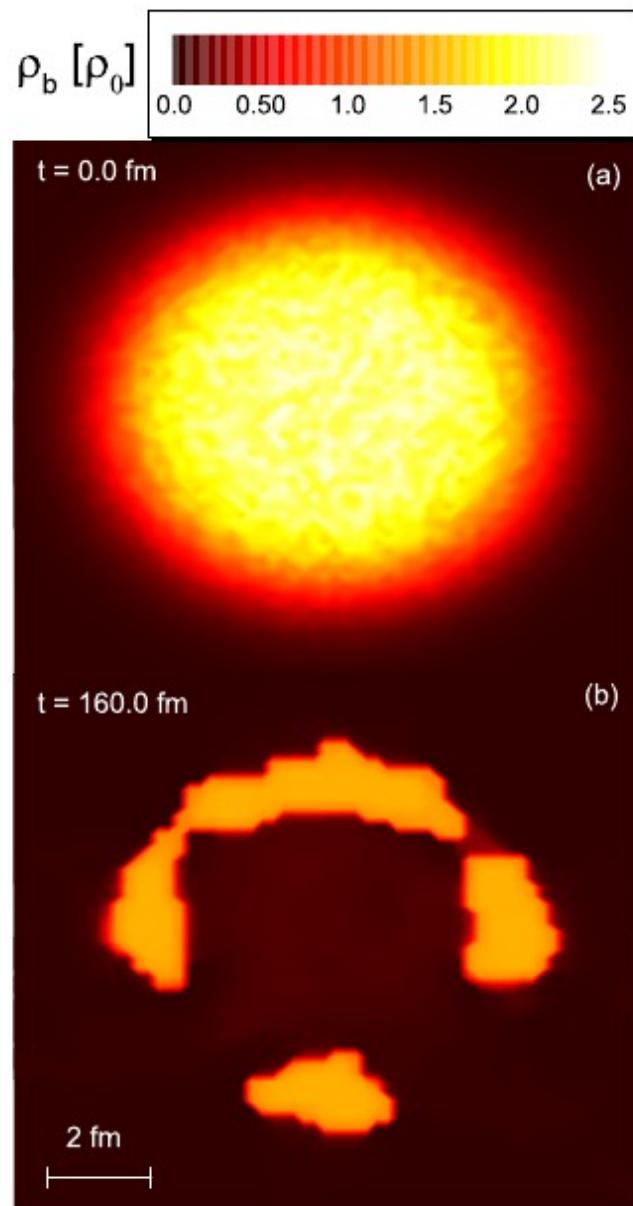
Time evolution



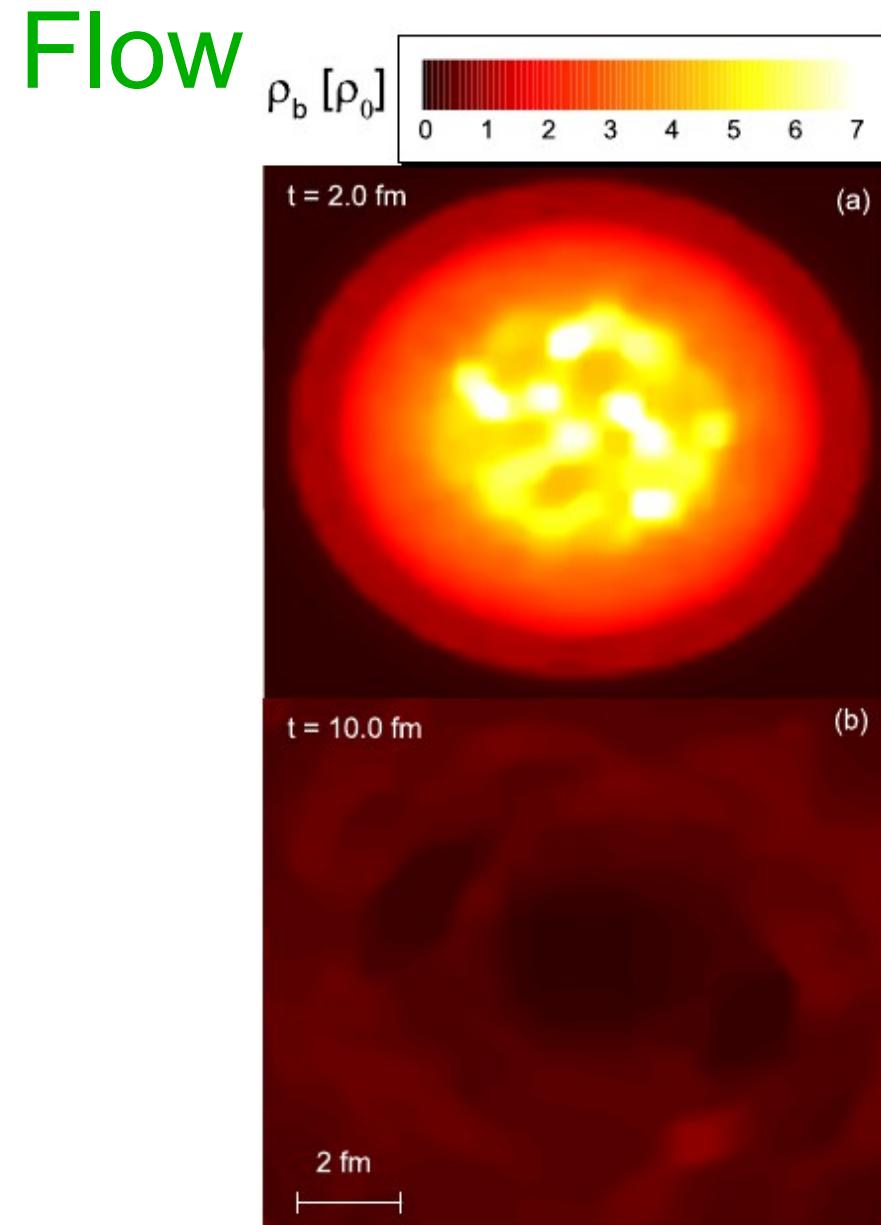
Oscillation of nearly stable droplets for “liquid-gas” EoS

Higher pressure leads to faster evolution of “QCD” EoS.

Steinheimer et al,
Phys.Rev. C89 (2014) 034901



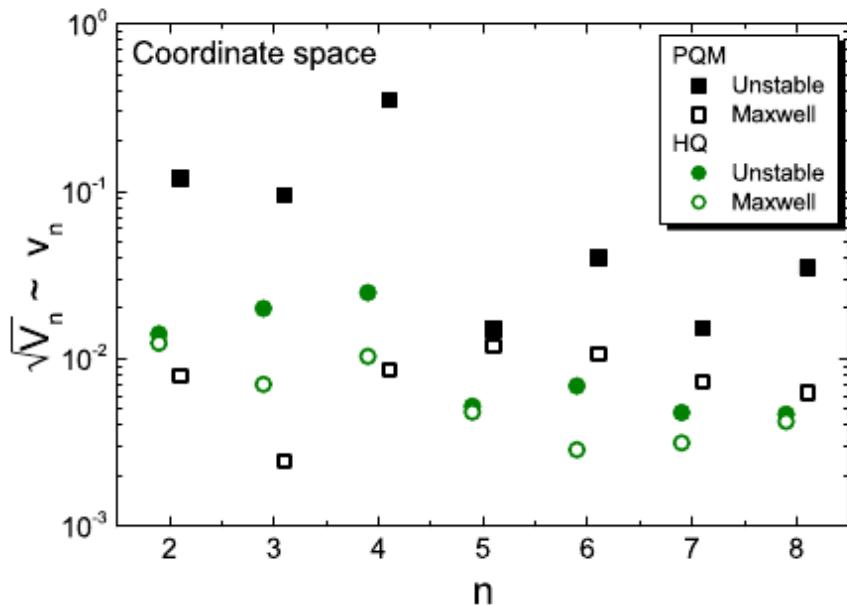
PQM (“liquid-gas”)



“QCD”

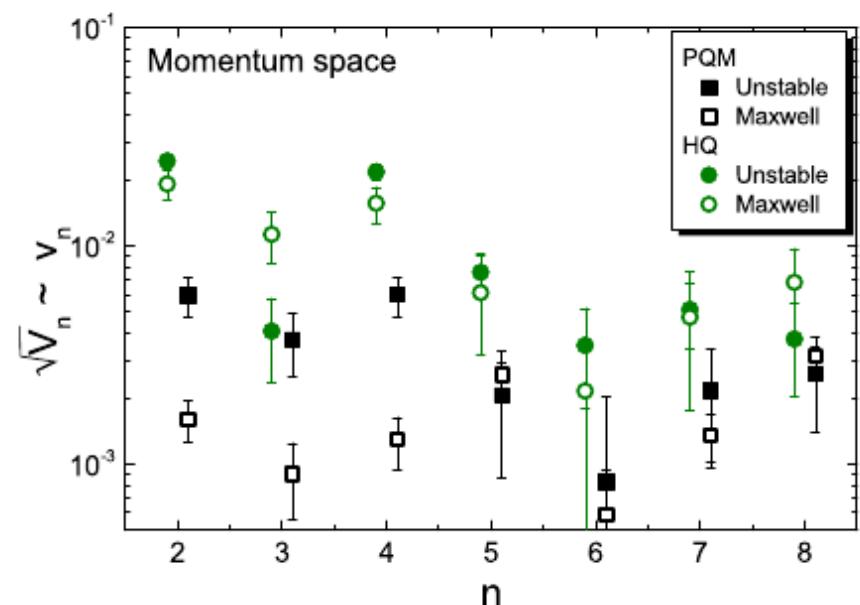
Flow

Coordinate space



Coordinate space asymmetries
sensitive to nearly stable droplet
formation in “liquid gas” EoS

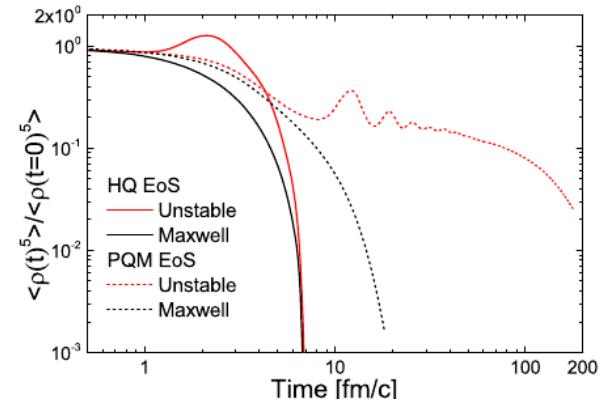
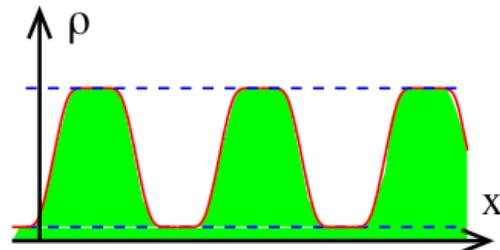
Momentum space



Small pressure of liquid:
Weak mapping into momentum space
hardly any effect of instabilities
In case of “QCD” EoS

Cluster a.k.a. nuclei

Even if total baryon number does
not fluctuate the baryon **density** does



Therefore measure production of NUCLEI: d, ³He, ⁴He, ⁷Li....

$$\langle d \rangle \sim \langle \rho_B^2 \rangle$$

$$\langle {}^3 He \rangle \sim \langle \rho_B^3 \rangle$$

$$\langle {}^7 Li \rangle \sim \langle \rho_B^7 \rangle$$

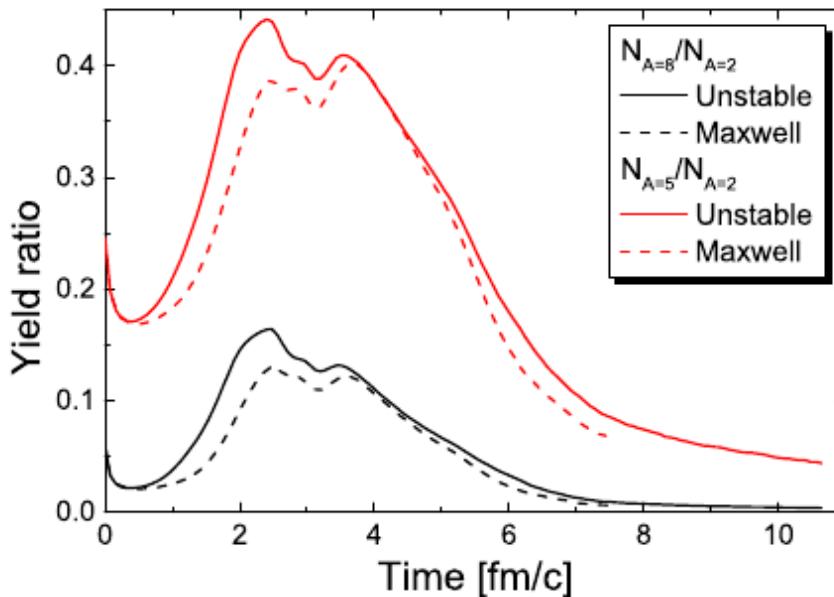
Extracts higher moments of the baryon **density** at freeze out

Nice Idea, but...

“Cluster” formation

“QCD” EoS

$$\left(\frac{S}{B}\right)_{\text{hadron-gas}} < \left(\frac{S}{B}\right)_{\text{QGP-liquid}}$$



Clumping in coordinate space is compensated by dilution on momentum space → tiny effect

Steinheimer et al,
Phys.Rev. C89 (2014) 034901

Back to



Higher moments (cumulants) and ξ

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp \{-\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots \right]. \quad \Rightarrow \quad \xi = m_\sigma^{-1}$$

- Moments (connected) of $q = 0$ mode $\sigma_V \equiv \int d^3x \sigma(x)$:

$$\kappa_2 = \langle \sigma_V^2 \rangle = VT\xi^2; \quad \kappa_3 = \langle \sigma_V^3 \rangle = 2VT^2\lambda_3\xi^6;$$

$$\kappa_4 = \langle \sigma_V^4 \rangle_c \equiv \langle \sigma_V^4 \rangle - 3\langle \sigma_V^2 \rangle^2 = 6VT^3 [2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

- Tree graphs. Each propagator gives ξ^2 .

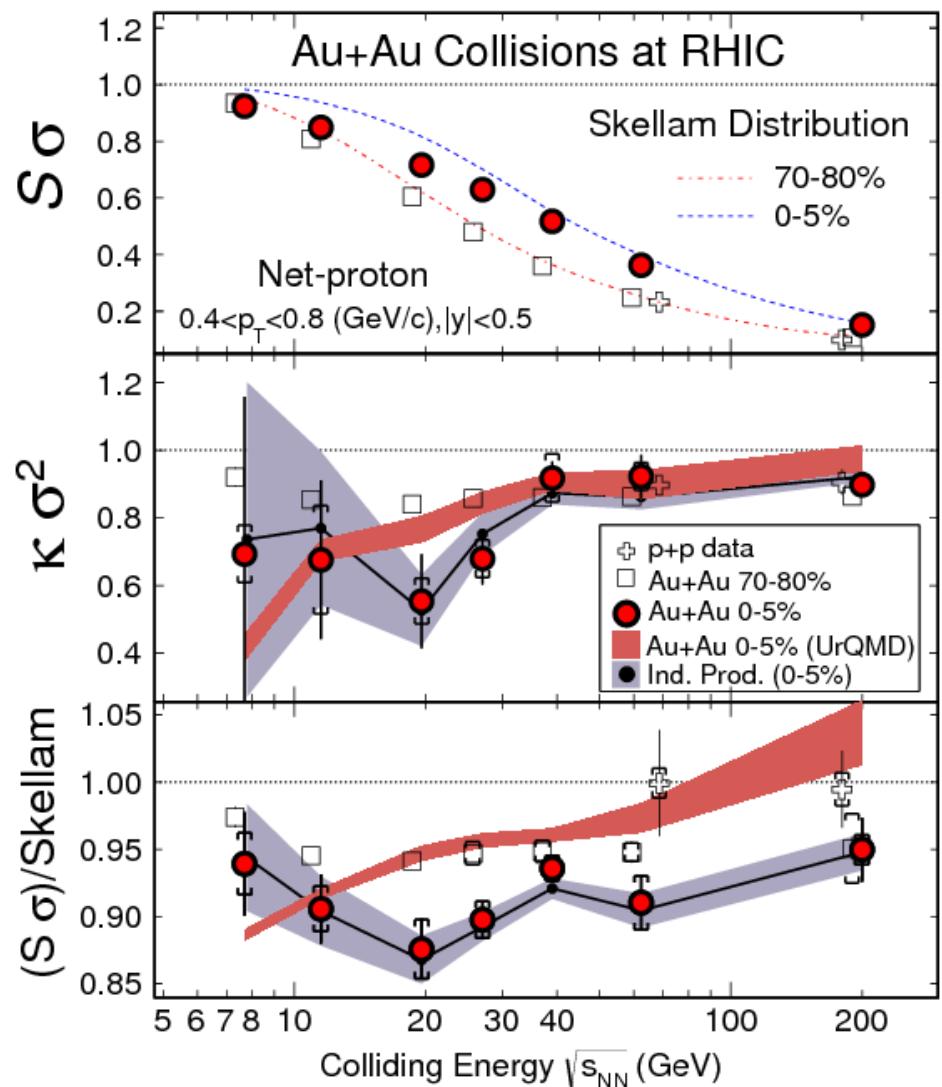
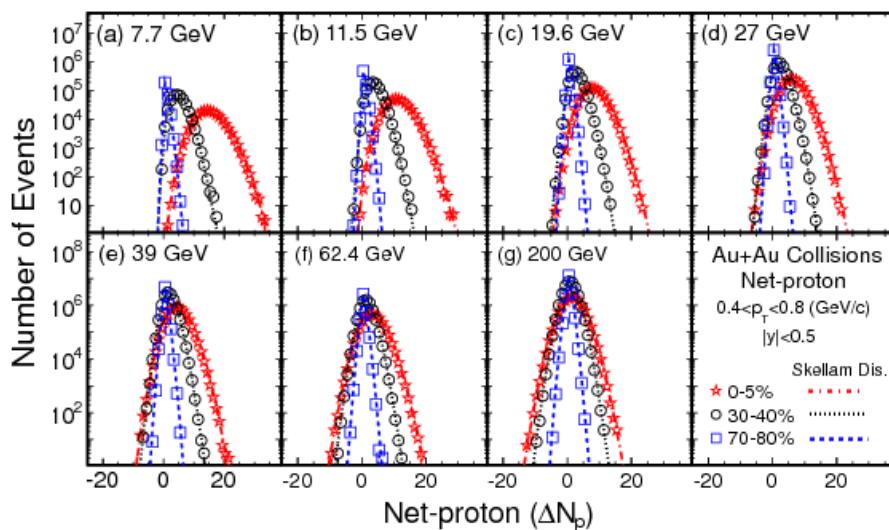


- Scaling requires “running”: $\lambda_3 = \tilde{\lambda}_3 T(T\xi)^{-3/2}$ and $\lambda_4 = \tilde{\lambda}_4(T\xi)^{-1}$, i.e.,

$$\kappa_3 = \langle \sigma_V^3 \rangle = 2VT^{3/2}\tilde{\lambda}_3 \xi^{4.5}; \quad \kappa_4 = 6VT^2 [2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4]\xi^7.$$

STAR net-proton cumulants

(Phys.Rev.Lett. 112 (2014) 032302)

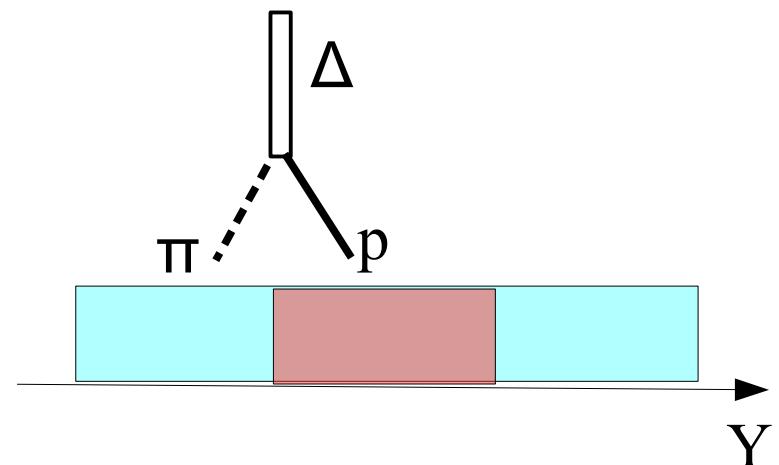
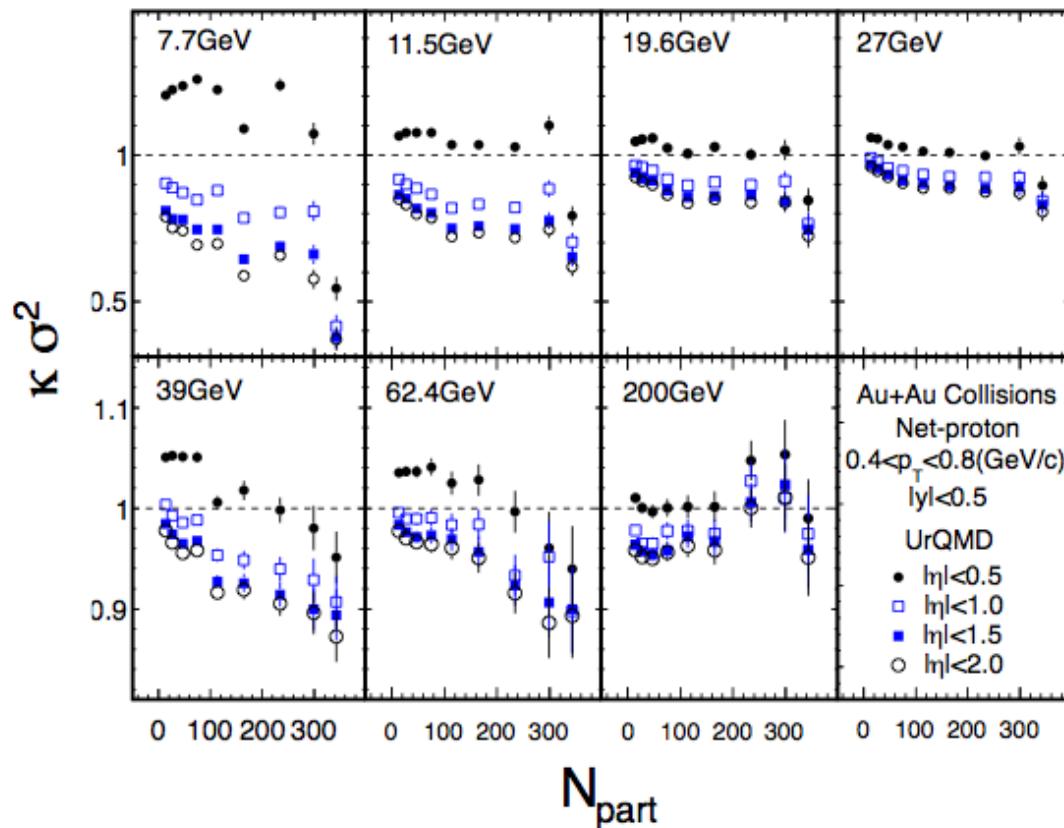


Things to consider

- Fluctuations of conserved charges ?!
- Higher cumulants probe the tails. Statistics!
- The detector “fluctuates” !
- Net-protons different from net-baryons
 - Isospin fluctuations
- Auto-correlations
- Beware of the “Poissonizer”

Auto Correlations

Luo et al, arXiv:1303.2332

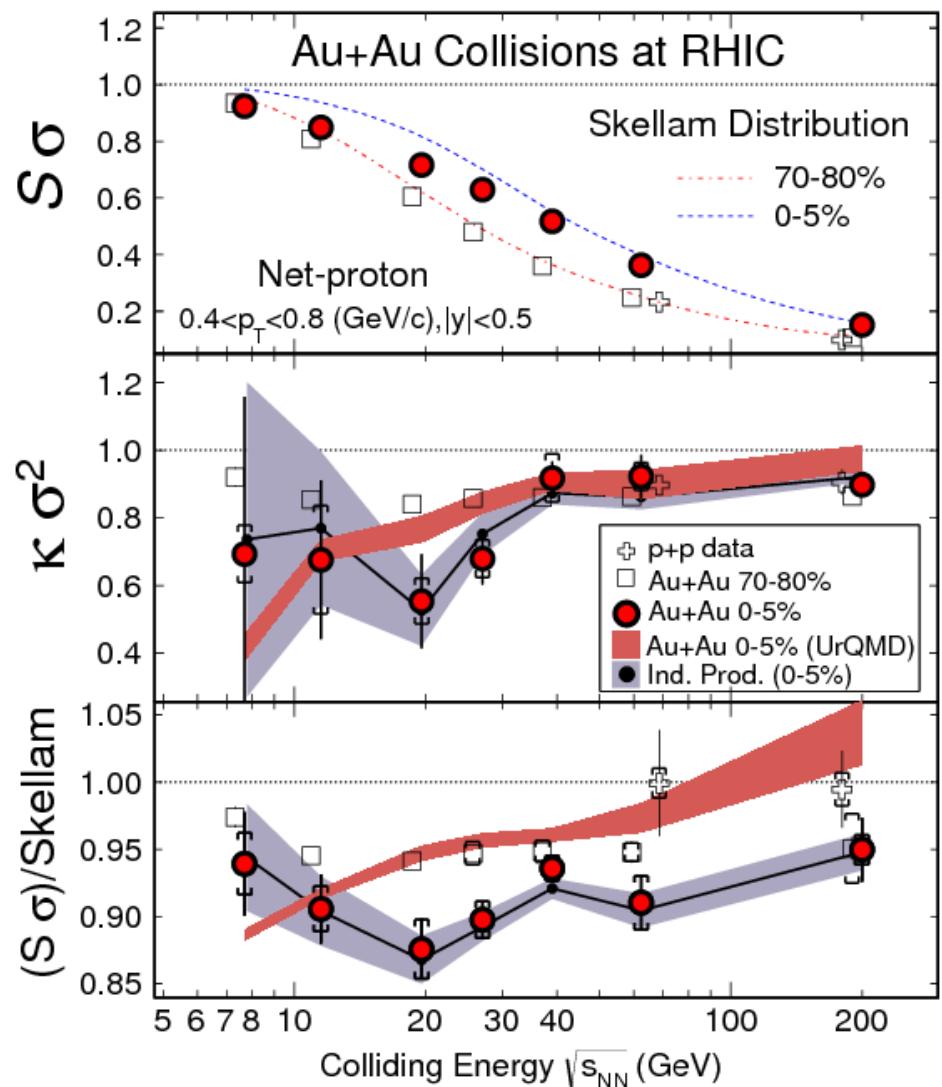
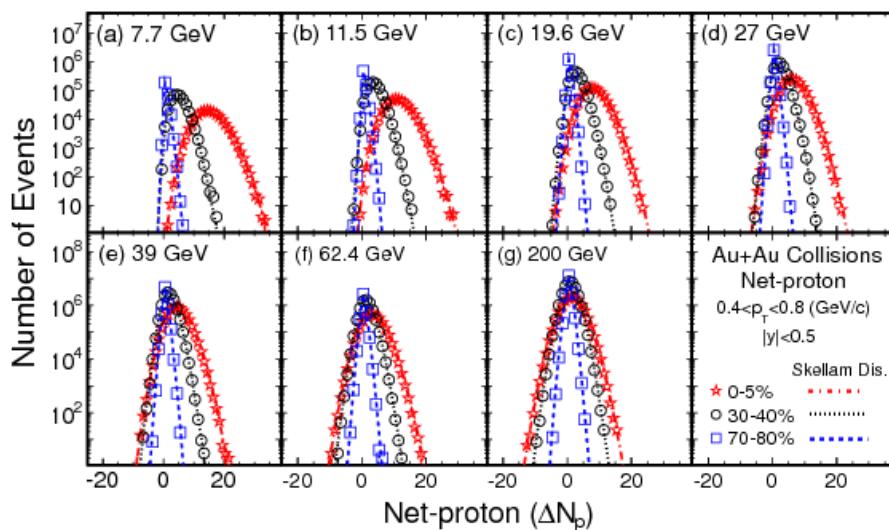


Strong correlation between multiplicity determination and proton cumulants
Due to baryon resonances

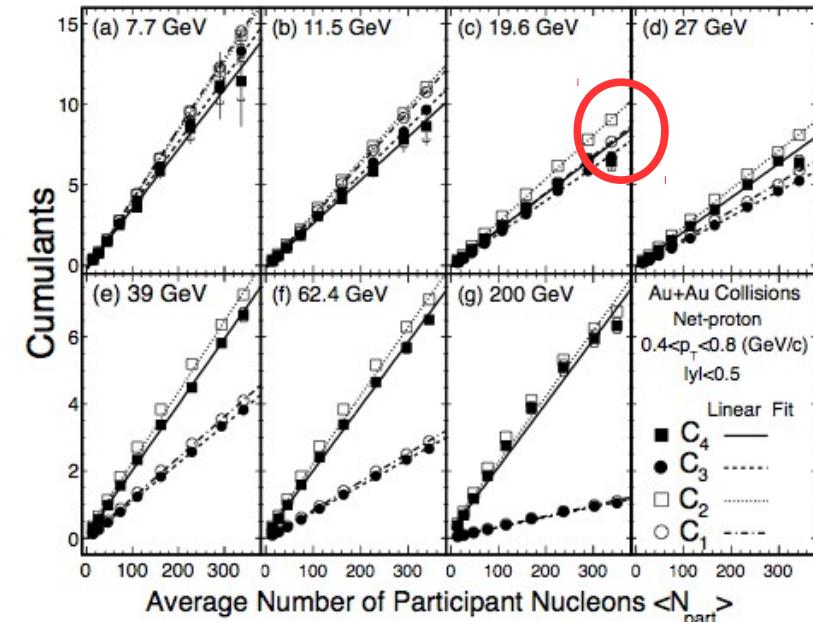
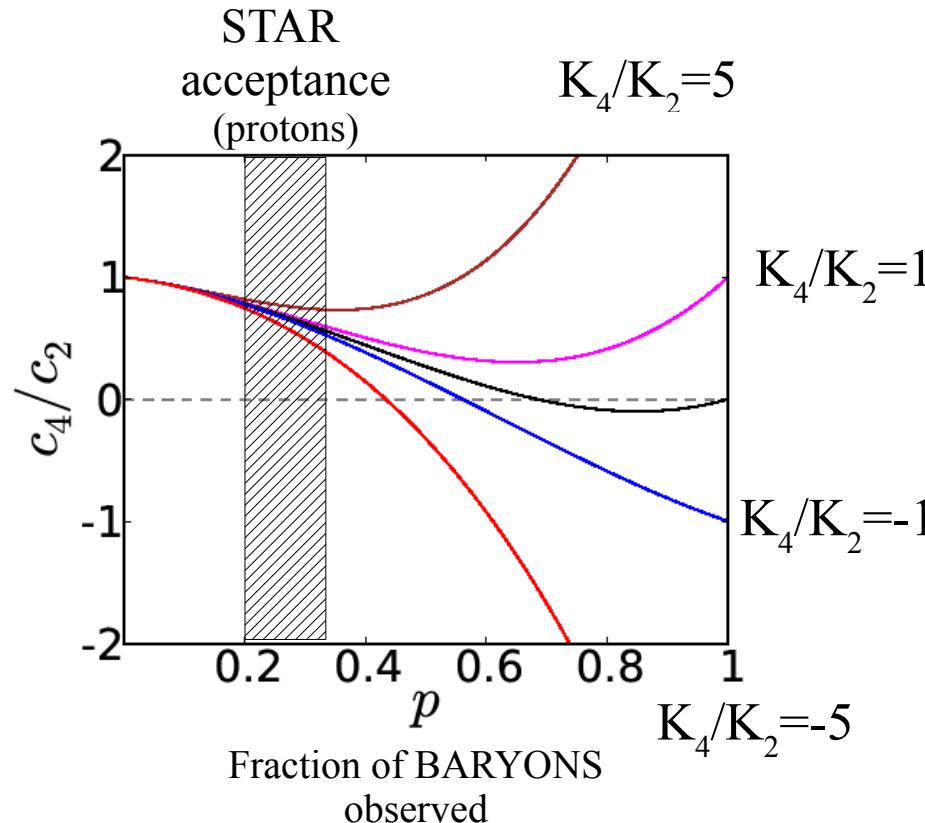
Need to determine multiplicity far away in rapidity from cumulants

STAR net-proton cumulants

(Phys.Rev.Lett. 112 (2014) 032302)



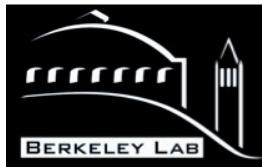
The “Poissonizer”



NA49 “sees” 32 protons per unit rapidity at top SPS energies!!!

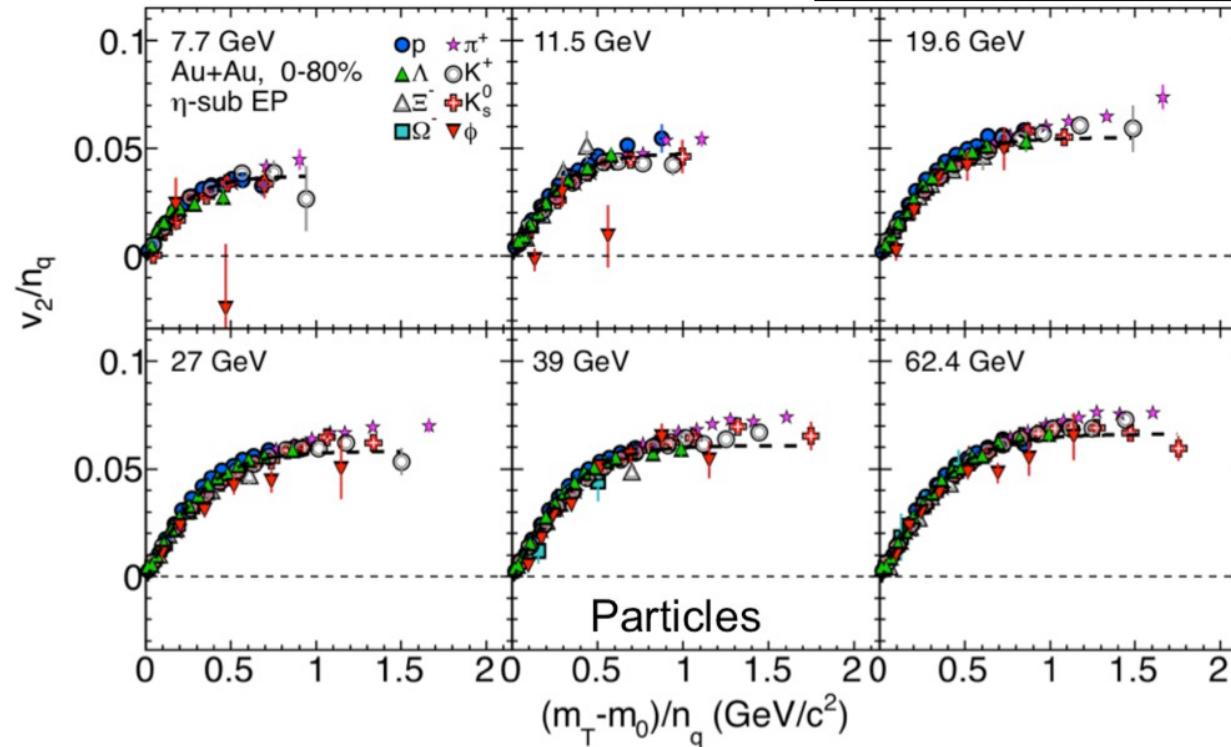
Flow

v_2 NCQ Scaling of Particles



Rihan Haque, Mo, 15:00

Phys. Rev. C 88, 014902 (2013)



- NCQ-scaling holds for particles and anti-particles separately at all energies
→ Partonic degrees of freedom?

- High $m_T - m_0$ not measured at lower energies
- Do ϕ -mesons deviate?

NCQ = Number of Constituent Quark

Particle and Anti-particle flow

Steinheimer et al Phys.Rev. C86 (2012) 044903 :

- Excitation function of v_2
- Centrality dependence of freeze out parameters

Both agree with STAR measurement

Essential: stopping of baryon number
 Explains difference in elliptic flow
 between protons and anti-protons.

Not yet included:
 Stopping of **isospin**.

Qualitatively explains the trend seen for pions

Strangeness conservation: strangeness chemical Potential same sign as baryon chemical potential:
 Flow difference of kaons same sign as protons

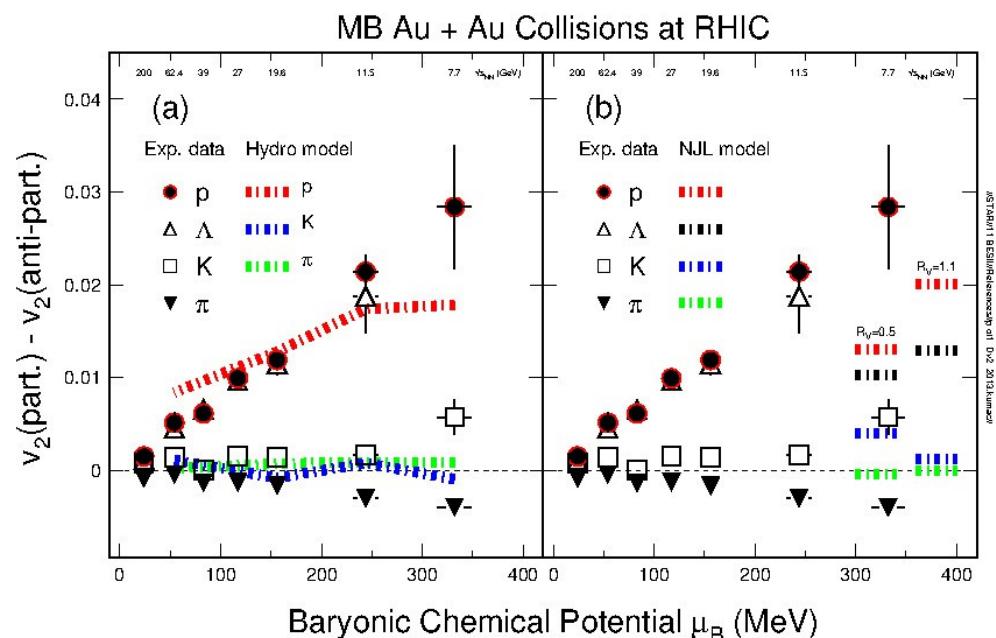
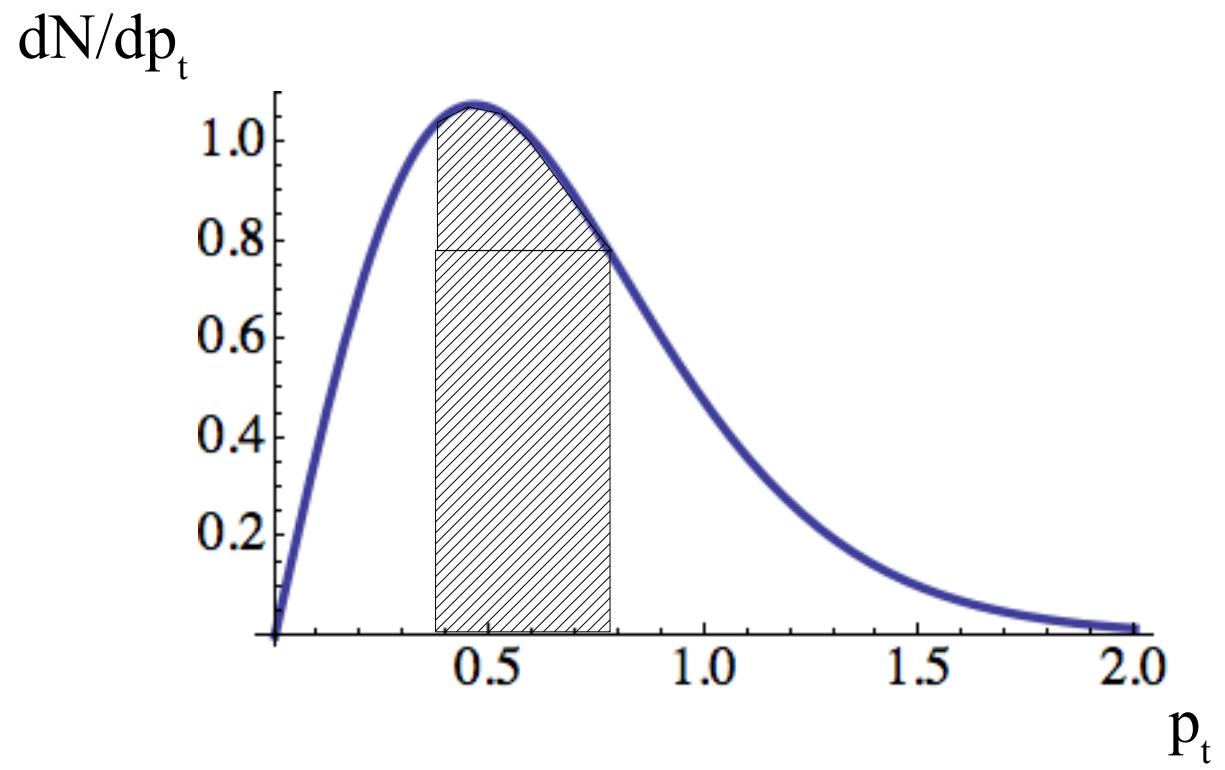


Figure courtesy N. Xu

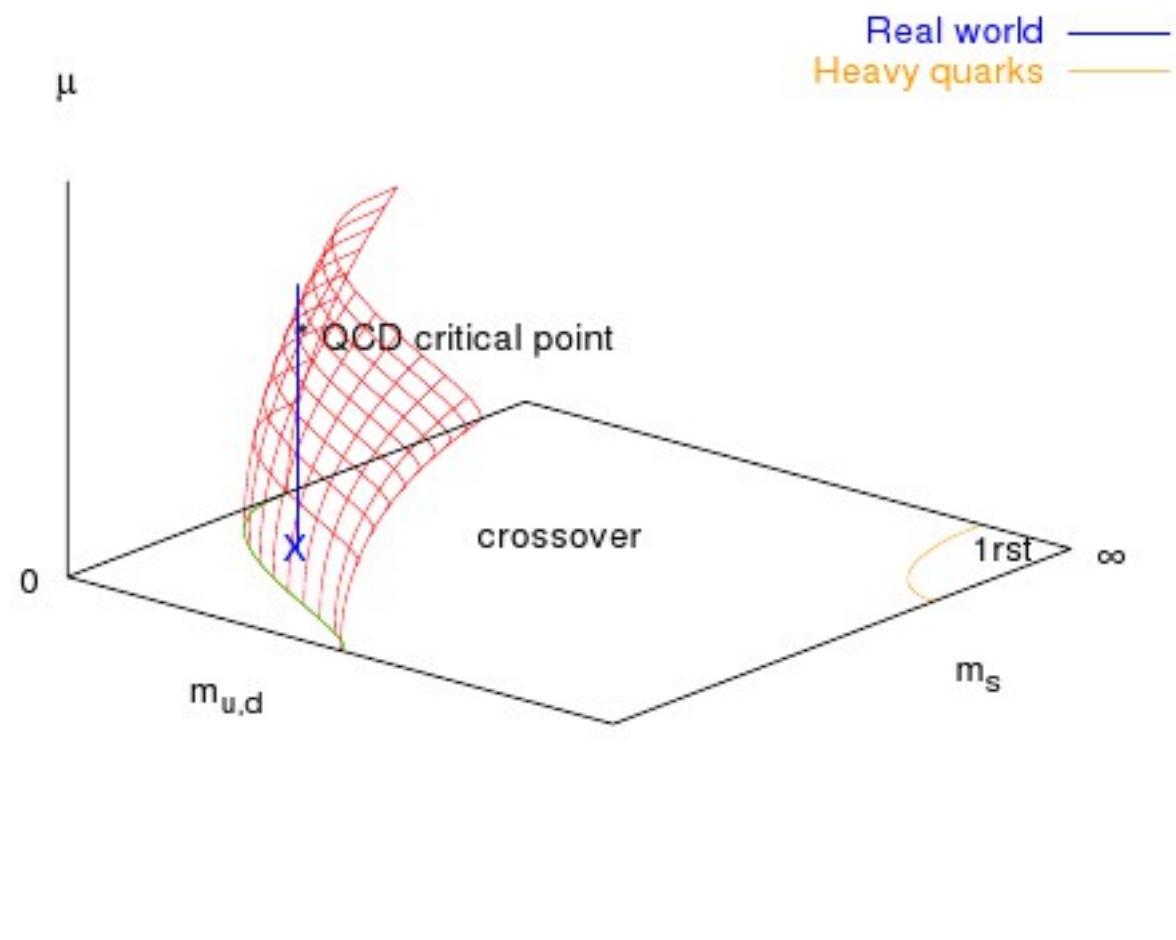
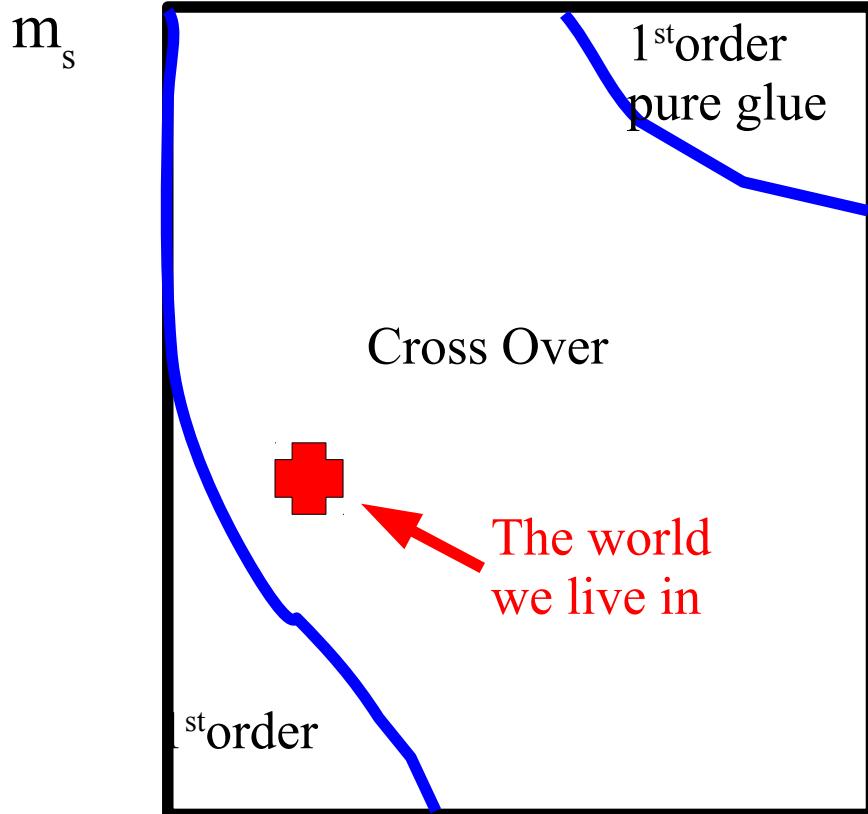
Summary

- Essential difference between liquid-gas and QCD EoS
 - Important phenomenological consequences
- Dynamical treatment of first order phase transition including instabilities within fluid dynamics
 - Good tool for testing observables
 - So far no good observable for instabilities and droplet formation
- Higher order cumulants: Not there yet!
 - Auto correlations
 - full thermal phase space
 - Statistics at low energy
- Quark number scaling: Not ruled out at low energy
 - Particle-anti particle v_2 : stopping!
 - Need a statistically significant phi measurement

BACKUP



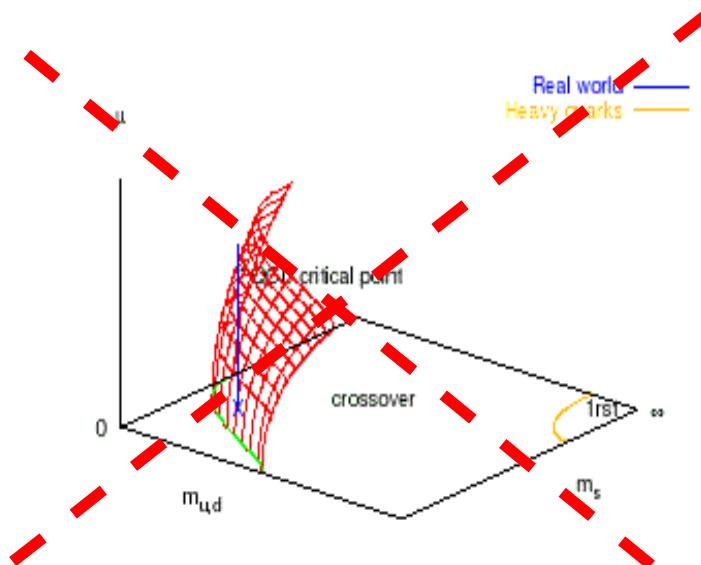
Is there a critical point?



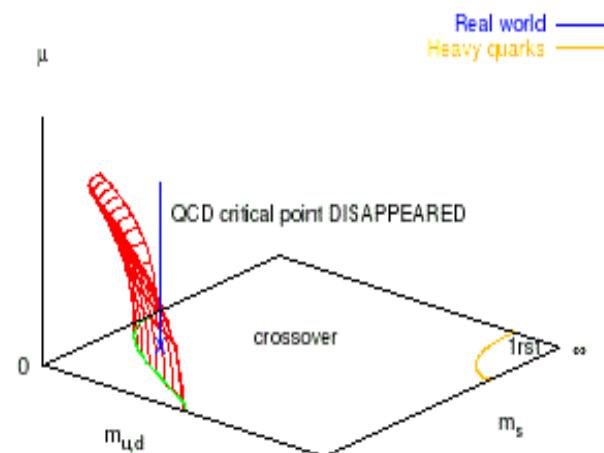
Lattice and the critical point

Forcrand, Philipsen

A non-standard scenario: no critical point?



$$\text{sign of } c_1 = \frac{dm_c(\mu)}{d\mu^2} \Big|_{\mu=0}$$

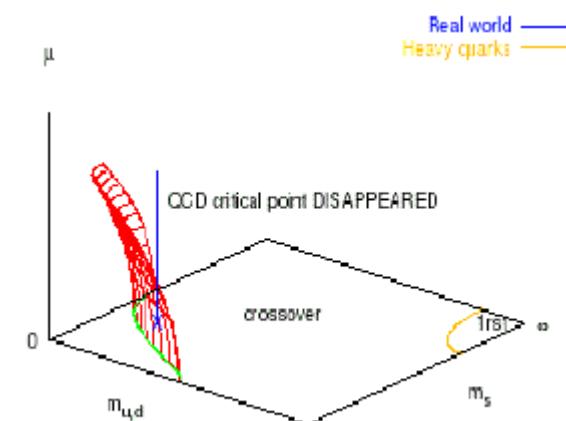
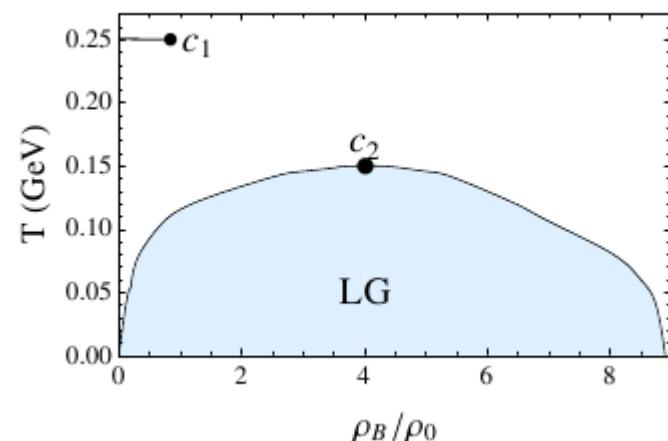
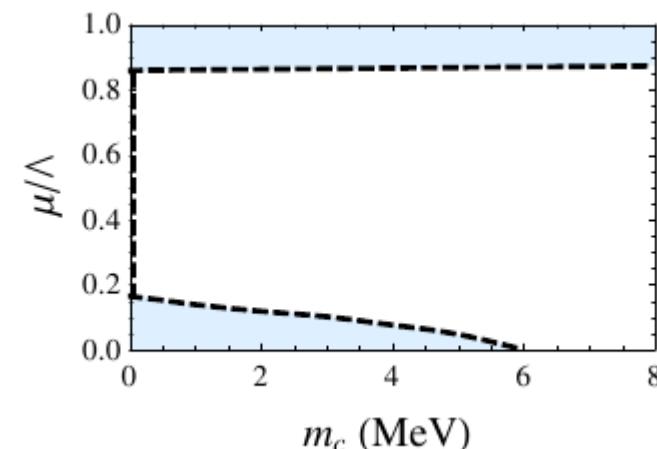
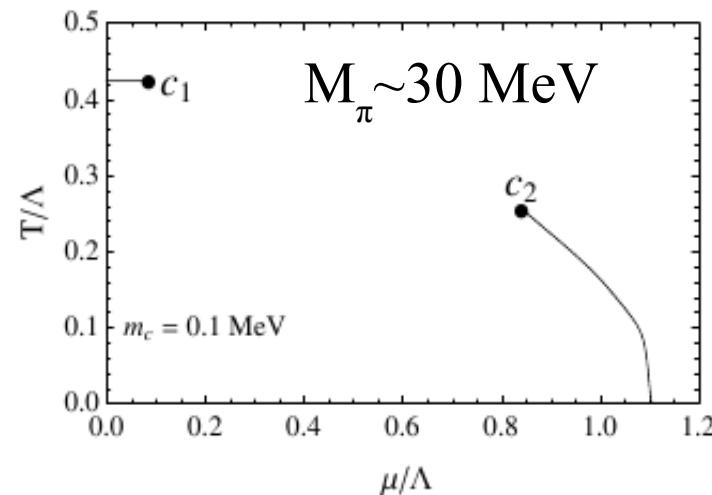


Favored by Lattice QCD

Note: Surface may bend back!!!

Two Critical Points ?!

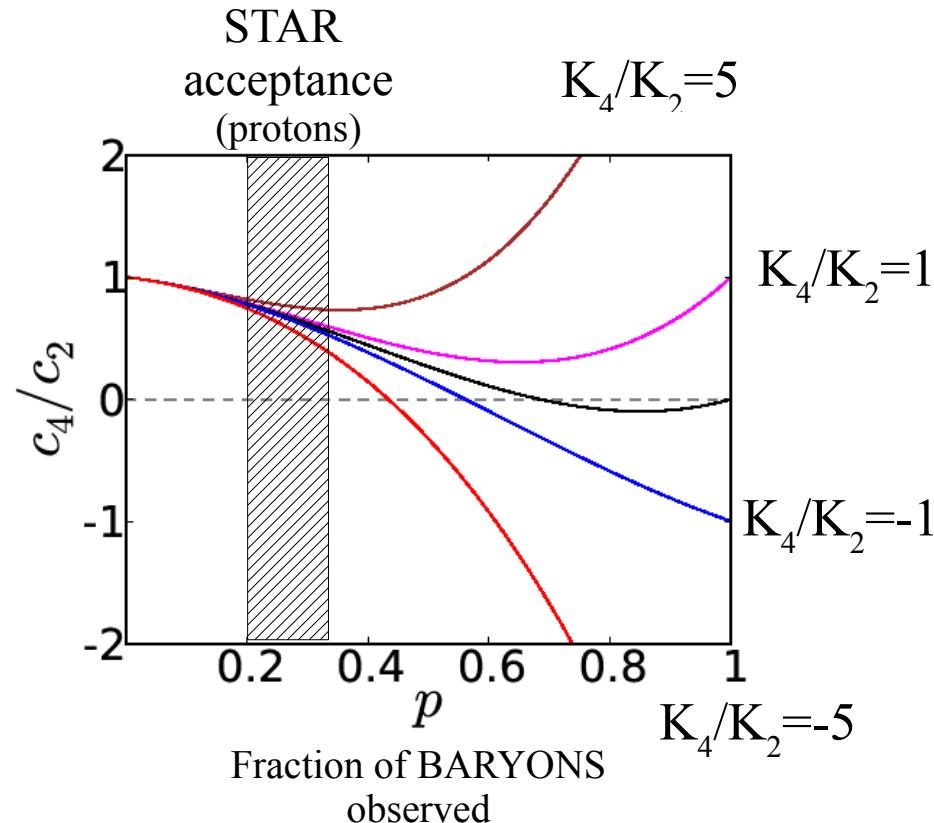
M. Pinto et al, Phys.Rev. C82 (2010) 055205



Seen in both Nambu and Linear Sigma Model

Finite Acceptance

(A. Bzdak and V.K., Phys.Rev. C86 (2012) 044904 and arXiv:1312.4574)



Spoils the fun for Baryon cumulants

Electric Charge cumulants better.
BUT issue with separation of (rapidity scales) at low energies

Baryon Number Conservation

A. Bzdak, V. Skokov, VK, Phys.Rev. C87 (2013) 014901



$p_B, p_{B_{\text{bar}}}$

$$P_B(n) = \left(\frac{p_B}{p_{\bar{B}}}\right)^{n/2} \left(\frac{1-p_B}{1-p_{\bar{B}}}\right)^{(B-n)/2} \quad (7)$$

$$\times \frac{I_n(2z\sqrt{p_B p_{\bar{B}}}) I_{B-n}(2z\sqrt{(1-p_B)(1-p_{\bar{B}})})}{I_B(2z)},$$

$$z = \sqrt{\langle N_B \rangle \langle N_{\bar{B}} \rangle}.$$

Protons only: $p_B = \frac{\langle n_B \rangle}{\langle N_B \rangle} \rightarrow \frac{\langle n_p \rangle}{\langle N_B \rangle}, \quad < 1/2$

Finite Acceptance

A. Bzdak, VK; Phys.Rev. C86 (2012) 044904

Effect of conservation laws get reduced with finite acceptance:
“Equilibration via ignorance”

Model with binomial distribution: $p_{1,2}$ = probability to see particle, antiparticle

True distribution

$$p(n_1, n_2) = \sum_{N_1=n_1}^{\infty} \sum_{N_2=n_2}^{\infty} P(N_1, N_2) \frac{N_1!}{n_1!(N_1 - n_1)!} p_1^{n_1} (1 - p_1)^{N_1 - n_1} \\ \times \frac{N_2!}{n_2!(N_2 - n_2)!} p_2^{n_2} (1 - p_2)^{N_2 - n_2}.$$

Finite Acceptance

True

$$F_{ik} \equiv \left\langle \frac{N_1!}{(N_1 - i)!} \frac{N_2!}{(N_2 - k)!} \right\rangle = \sum_{N_1=i}^{\infty} \sum_{N_2=k}^{\infty} P(N_1, N_2) \frac{N_1!}{(N_1 - i)!} \frac{N_2!}{(N_2 - k)!},$$

Measured

$$f_{ik} \equiv \left\langle \frac{n_1!}{(n_1 - i)!} \frac{n_2!}{(n_2 - k)!} \right\rangle = \sum_{n_1=i}^{\infty} \sum_{n_2=k}^{\infty} p(n_1, n_2) \frac{n_1!}{(n_1 - i)!} \frac{n_2!}{(n_2 - k)!}.$$

$$f_{ik} = p_1^i \cdot p_2^k \cdot F_{ik}.$$

$$c_1 = pK_1,$$

$$c_2 = p(1-p)\underline{N} + p^2K_2,$$

$$c_3 = p(1-p^2)K_1 + 3p^2(1-p)(\underline{F_{20}} - \underline{F_{02}} - NK_1) + p^3K_3,$$

$$N = N_1 + N_2$$

$$\begin{aligned} c_4 = & Np(1-p) - 3N^2p^2(1-p)^2 + 6p^2(1-p)(\underline{F_{02}} + \underline{F_{20}}) - 12K_1p^3(1-p)(\underline{F_{20}} - \underline{F_{02}}) \\ & + 6Np^3(1-p)(K_1^2 - K_2) + p^2(1-p^2)(K_2 - 3K_1^2) \\ & + 6p^3(1-p)(\underline{F_{03}} - \underline{F_{12}} + \underline{F_{02}} + \underline{F_{20}} - \underline{F_{21}} + \underline{F_{30}}) + p^4K_4. \end{aligned}$$

Due to “acceptance” not only Cumulants of the true distribution enter

Finite Acceptance

Unfolding?
Can be done (in principle)

$$pK_1 = c_1,$$

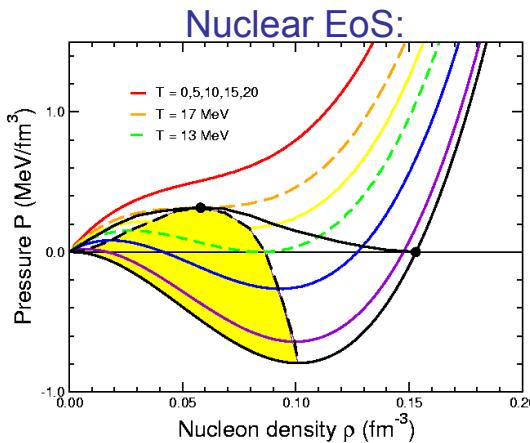
$$p^2 K_2 = c_2 - \underline{n}(1-p),$$

$$p^3 K_3 = c_3 - c_1(1-p^2) - 3(1-p)(\underline{f_{20}} - \underline{f_{02}} - nc_1),$$

$$\begin{aligned} p^4 K_4 = & c_4 - np^2(1-p) - 3n^2(1-p)^2 - 6p(1-p)(\underline{f_{20}} + \underline{f_{02}}) + 12c_1(1-p)(\underline{f_{20}} - \underline{f_{02}}) \\ & -(1-p^2)(c_2 - 3c_1^2) - 6n(1-p)(c_1^2 - c_2) \\ & - 6(1-p)(\underline{f_{03}} - \underline{f_{12}} + \underline{f_{02}} + \underline{f_{20}} - \underline{f_{21}} + \underline{f_{30}}). \end{aligned}$$

Requires measurement of factorial moments, $f_{20}, f_{02}, f_{21}, \dots$ with
GOOD precision

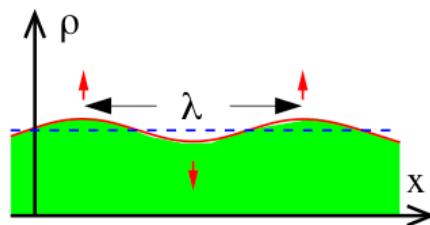
Spinodal Multifragmentation



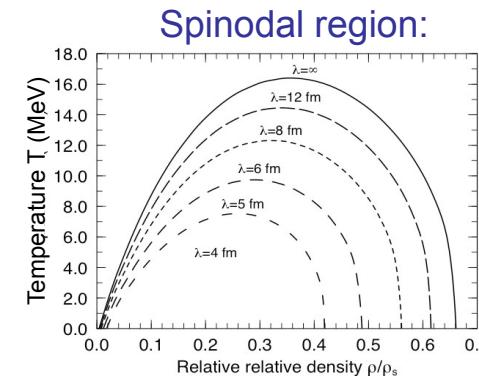
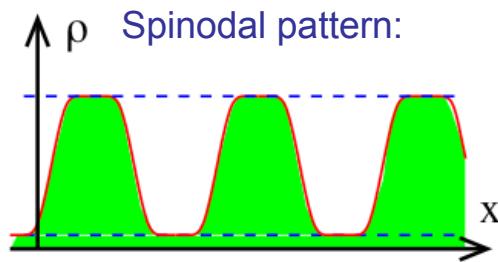
1st order phase transition



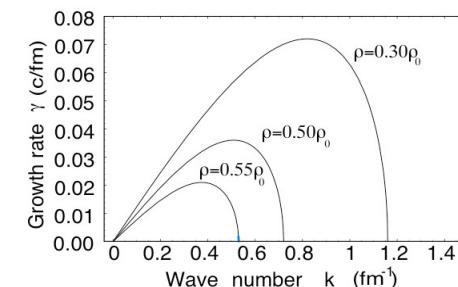
Spinodal instability



Density undulations
may be amplified



Growth rates:



Fragments
 \approx equal!

Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
Physics Reports 389 (2004) 263



Highly non-statistical => Good candidate signature

CLUMPING of Baryon Density

J. Randrup

Input required for realistic estimate of conservation effects

Note: This is likely only to work at lower energies where we have baryon stopping

Note: at low energies anti-protons likely to be irrelevant

Need:

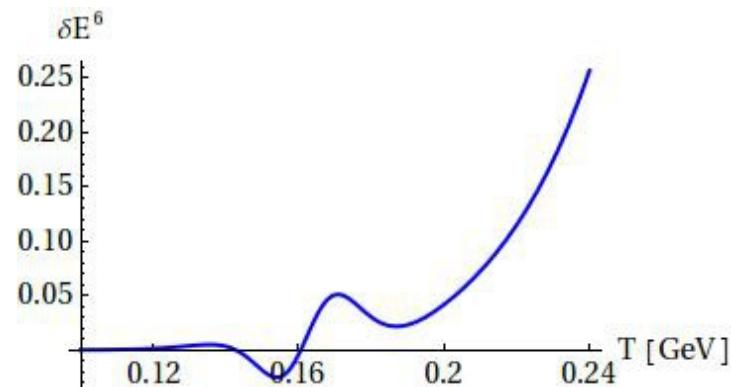
- Total number of protons and (anti-protons) (4π)
- Number of protons and (anti-protons) actually measured
- Total number of charged particles

Big Question: Over what rapidity range are the various charges conserved?

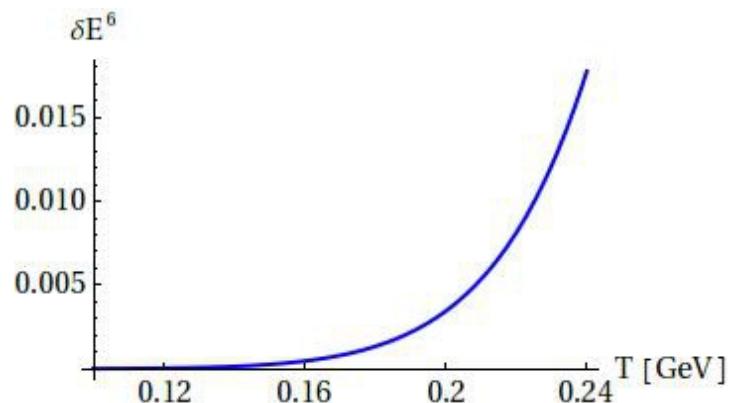
- Balance Functions? Only averages!

QCD vs HRG

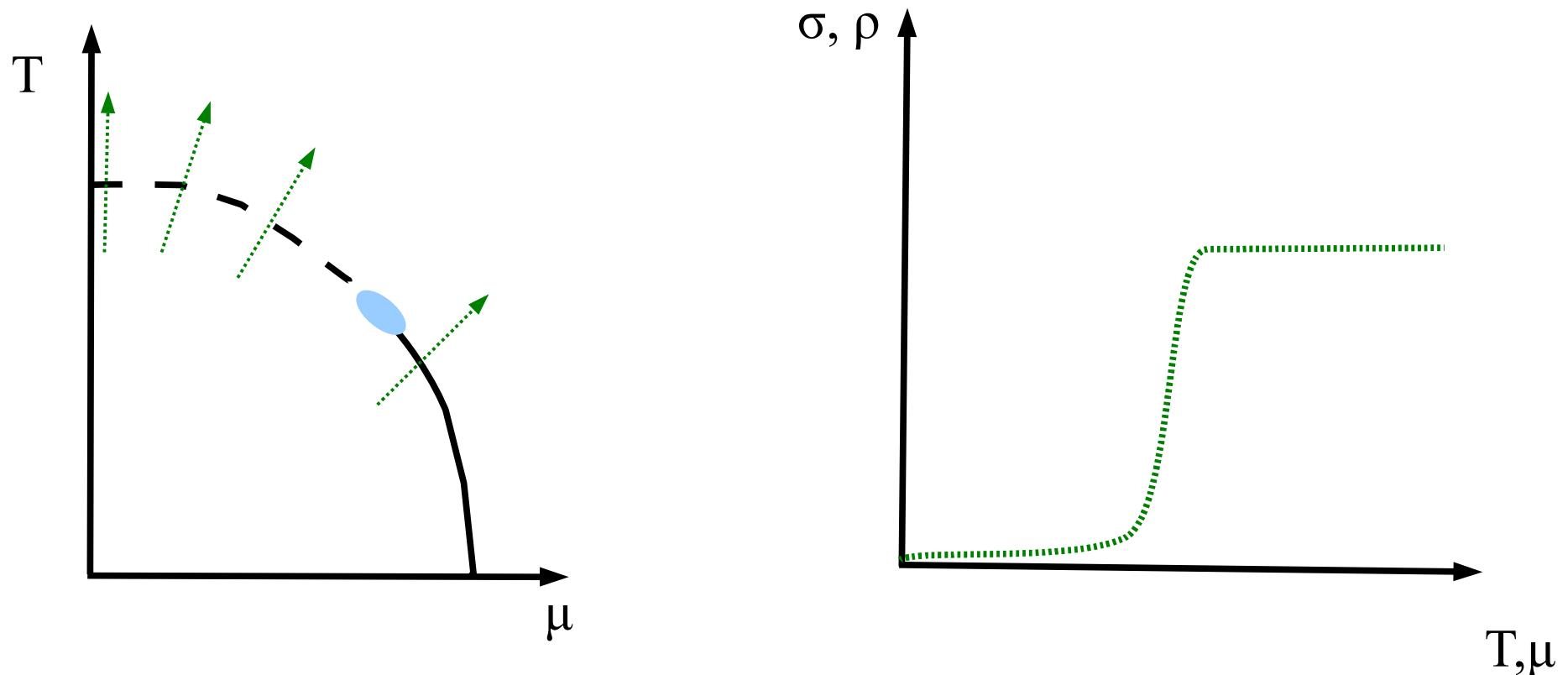
QCD



“HRG”



Generic Phase Diagram

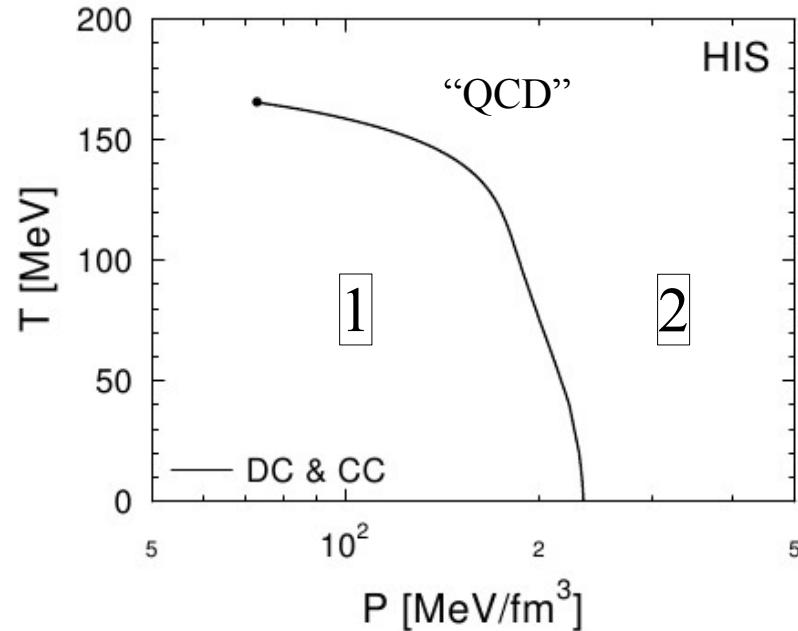
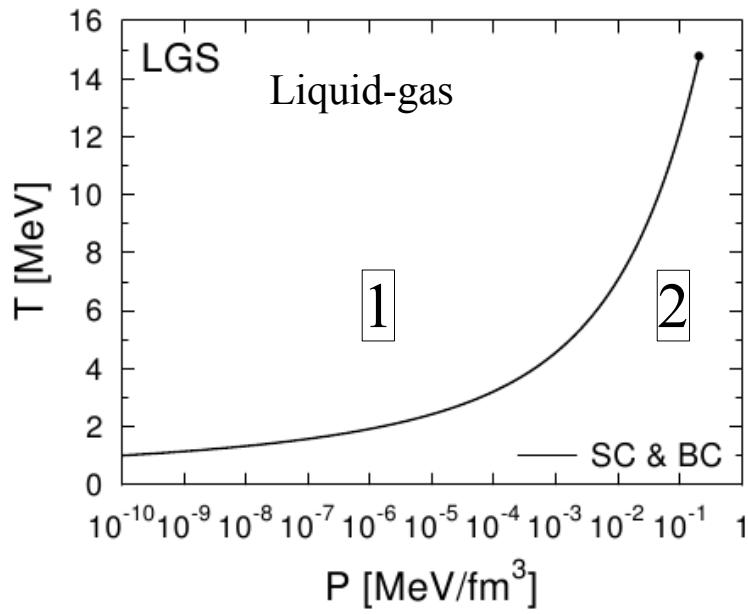


“Simply” use appropriate combination of T and μ

Requires: $\langle (\delta E)^n \rangle$ $\langle (\delta N_B)^n \rangle$ $\langle (\delta E)^m (\delta N_B)^n \rangle$ Mixed cumulants!

Difference between Liquid Gas and QCD PT

Dexheimer et al, arXiv:1302.2835



Clausius-Clapeyron: $\frac{dP}{dT} = \frac{S_1/B_1 - S_2/B_2}{1/\rho_1 - 1/\rho_2}$

$\rho_2 > \rho_1 \rightarrow (1/\rho_1 - 1/\rho_2) > 0$

$$\frac{dP}{dT} > 0 \rightarrow S_1/B_1 > S_2/B_2$$

$$\left(\frac{S}{B}\right)_{gas} > \left(\frac{S}{B}\right)_{liquid}$$

$$\frac{dP}{dT} > 0 \rightarrow S_1/B_1 < S_2/B_2$$

$$\left(\frac{S}{B}\right)_{hadron-gas} < \left(\frac{S}{B}\right)_{QGP-liquid}$$